Composite Fermion Theory of Fractional Chern Insulators

Luiz H. Santos

Chaos, Duality and Topology in Condensed Matter Theory

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Ramanjit Sohal    Eduardo Fradkin
Also:
Tang et al., 2011
Sun et al., 2011
Sheng et al., 2011
Liu et al., 2012
Lauchli et al., 2013, ...
The “Conventional” Quantum Hall Effect

- Two dimensional electron gas
- External perpendicular magnetic field

\[ \ell_B = \sqrt{\frac{\hbar c}{e B}} \]

\[ \ell_B \approx 250\,\text{Å} \quad (B = 1\,\text{T}) \]

\[ \ell_B \gg a \]

Magnetic length much larger than lattice spacing

Stormer, RMP 1999
Landau levels

\[ H = \sum_j \frac{1}{2m} \left( \vec{p}_j - \vec{A}(\vec{r}_j) \right)^2 + \sum_{i,j} V(\vec{r}_i - \vec{r}_j) \]

LLL single particle wave functions

\[ \psi_{0,m}(z = x + iy) \propto z^m e^{-\frac{|z|^2}{4\ell_B^2}} \]

\[ \Psi_{\nu=1/m}(z_1, \ldots, z_N) \propto \prod_{i \neq j} (z_i - z_j)^m \prod_i e^{-\frac{|z_i|^2}{4\ell_B^2}} \quad m = \text{odd} \]

Laughlin 1983
Jain’s composite fermion picture

\[ \Psi_{\nu=1/m}(z_1, \ldots, z_N) \propto \prod_{i \neq j} (z_i - z_j)^m \prod_i e^{-\frac{|z|^2}{4 \ell_B^2}} = \prod_{i \neq j} (z_i - z_j)^{m-1} \Psi_{\nu=1}(z_1, \ldots, z_N) \]

Each electron (on average) becomes bound to \((m-1)\) flux quanta forming a composite fermion (CF). CFs fully fill a Landau level.
Jain’s composite fermion picture

J. K. Jain, 1989

More generally, if each electron screens $2k$ flux quanta, it can form a gapped state if the composite fermion occupies $p$ Landau levels:

$$N_{\phi}^{\text{eff}} = N_{\phi} - 2k N_e$$

$$\nu = \frac{p}{2k p + 1}$$

For $k = 1$:

$$\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \ldots$$
Fractional Chern Insulators

- FQH states in partially filled Chern bands
- Strong TRS breaking and lattice effects

What kinds of fractional topological states can a “Chern lattice” support?

How to characterize these states in terms of lattice filling and Hall conductance?

These questions will be addressed via composite fermion states.

Implementation of the flux attachment via discretized Chern-Simons theory.
Discretized Chern-Simons Action

\[ N_v = N_f \]

Local vertex-face correspondence

\[ S = \frac{k}{2\pi} \int dt \left[ A_v M_{v,f} \Phi_f - \frac{1}{2} A_e K_{e,e'} \dot{A}_{e'} \right] \quad k \in \mathbb{Z} \]

Square

Kagome

Dice

\[ q_v = \frac{k}{2\pi} M_{v,f} \Phi_f \]

Charge-flux attachment

\[ [A_e, A_{e'}] = i \frac{2\pi}{k} K_{e',e}^{-1} \]

See also: Frohlich and Marchetti 1988, Fradkin 1989, Eliezer and Semenoff 1992
Kagome Chern Insulator

\[ H_0 = - \sum_{\langle x, x' \rangle} t e^{i \phi_{x,x'}} \psi^\dagger(x) \psi(x') + \text{H.c.} \]

What are candidate FCI states when the lowest Chern band is partially filled?

\[ C = -1 \]
\[ C = 0 \]
\[ C = 1 \]

\[ \phi_+ = \phi_- = \pi/2 \]

D. Green, LHS, C. Chamon (2010)
Flux Attachment on the kagome lattice*

\[ n^\alpha(x) = \theta \Phi^\alpha(x) \]
\[ \theta = \frac{1}{2\pi(2k)} \]

\[ S = S_F + S_{CS} + S_{int} \]

\[ S_F[\psi, \psi^\dagger, A_\mu] = \int_t \sum_x \psi^\dagger(x, t)(iD_0 + \mu)\psi(x, t) \]
\[ - \int_t \sum_{\langle x, x' \rangle} (\psi^\dagger(x, t)e^{i(A_j(x,t) + \phi_{x,x'})}\psi(x', t) + h.c.), \]

* System on a disk

R. Sohal, LHS, E. Fradkin, arxiv: 1707.06118
Jain states

Mean-Field Hofstadter bands

\[ \sigma_{xy} = n_L \]

\[ n_L = \frac{3}{2\pi 2k_c} \phi \]

\[ \sigma_{xy} = \frac{C}{2kC + 1} \]

\[ C = \frac{1}{2\pi} \sum_{n \text{ filled}} \int_{BZ} d^2k f_{12}^n(k) \]

\[ \phi_+ = \phi_- = \pi/2 \quad k = 1 \]
Strong lattice effects → New Fractional States

Mean-Field Hofstadter bands

\[ \sigma_{xy} \neq n_L \]

\[ \sigma_{xy} = \frac{C}{2kC + 1} \]

\[ \phi_+ = \phi_- = \pi/2 \quad k = 1 \]
Effective Field Theory

\[ \mathcal{L} = \frac{K_{IJ}}{4\pi} \epsilon^{\mu\nu\lambda} a_I^\mu \partial_\nu a_J^\lambda - \frac{q_I}{2\pi} \epsilon^{\mu\nu\lambda} \tilde{A}_\mu \partial_\nu a_I^\lambda + l_I j_{qp}^\mu a_I^\mu \]

\[ K_{IJ} = \begin{pmatrix} -2k & 1 & 0 \\ 1 & C & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad q_I = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad a_I^\mu = \begin{pmatrix} B_I^\mu \\ A_I^\mu \\ C_I^\mu \end{pmatrix} \]

\[ \sigma_{xy} = -q^T K^{-1} q = \frac{C}{2kC + 1} \]

\[ Q_1 = -l^T K^{-1} q, \quad \theta_{l'} = -2\pi l^T K^{-1} l' \]

\[ \text{GSD} = |\det(K)| = |2kC + 1| \quad \text{(Torus)} \]
Translation Symmetry Fractionalization

\[
(T_2^a)^{-1}(T_1^a)^{-1}T_2^a T_1^a = e^{i\theta_{a,b}}
\]

Essin and Hermele, PRB 2013
Barkeshli et al., 1410.4540
Cheng et al, PRX 2016
Lu, Ran, Oshikawa, 1705.09298

\[ q_b = n_L \]
Ground State Manifold Momentum

\[ \sigma_{xy} = \frac{C}{(2kC+1)} \]

\[ n_L = \frac{r}{(2kC+1)} \]

<table>
<thead>
<tr>
<th>( n_L )</th>
<th>( \sigma_{xy} )</th>
<th>( (T_2^\phi)^{-1}(T_1^\phi)^{-1}T_2^\phi T_1^\phi )</th>
<th>( \hat{k}_1 \mid \phi \rangle )</th>
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<td>1/7</td>
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<td>-2 \frac{2\pi}{7} N_2</td>
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Summary

- A composite fermion theory of FCIs is studied using a discretized Chern-Simons theory on the kagome lattice.

- Fractional states whose filling do not match the Hall conductance identified at the mean-field theory, whose properties could probed by numerical studies and, hopefully, experiments.

- **Future directions:** extending this approach to other lattices; exploring the interplay of topological and symmetry broken phases, etc.
Thank you!