Supersymmetric Mirror Duality and Half-filled Landau level

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Planar electrons moving in transverse magnetic field

\[ B \]

\[ n_e \]

\[ \nu = \frac{n_e}{B/2\pi} \]

Exhibits a gap, with quantized hall conductivities when

\[ \nu \in \text{Integer} : \text{IQHE: understood via single-particle dynamics} \]

\[ \nu \in \text{Fractions} : \text{FQHE: emergent collective phenomena} \]

E.g. Laughlin states, hierarchical extensions, etc
Introduction/Motivation

At Half-filling, a metallic state (R. Willett, et al, 1993):

Composite fermion theory (Jain 89; Halperin, Lee, Read, 92):

\[
\nu = \frac{p}{2p + 1}
\]

When not exact half-filled, CF feels an reduced magnetic field \( \delta B = B - 4\pi n_e \)

Jain’s sequence \( \tilde{\nu} = p \)

Pairing instabilities of CF fermi surface: Moore-Read Pfaffian states (at \( \nu = 5/2 \))
Introduction/Motivation

At Half-filling, a metallic state (R. Willettt, et al, 1993):

Composite fermion theory (Jain 89; Halperin, Lee, Read, 92):

Issue: particle/hole (PH) symmetry

approximate symmetry at half-fillings, exact in the limit $\omega_c \gg E_{\text{int}}$

LLL:

PH pairs of Jain states have different CF description
Introduction/Motivation

New proposal: duality (Son, 14)

\[ L_{el} = i \bar{\Psi}_{el} \gamma^\mu (\partial_\mu + i A_\mu) \Psi_{el} \]

\[ L_{CF} = i \bar{\psi}_{cf} \gamma^\mu (\partial_\mu + 2i a_\mu) \psi_{cf} + \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda \]

no Chern-simons term for emergent gauge field

CF dirac fermion, map to itself under P-H symmetry

B.F. coupling (EM coupled to vortices)

background magnetic field

\[ \hat{B} = \frac{1}{2} \epsilon^{ij} \partial_i \hat{A}_j \]

density of composite fermion:

\[ \rho_{cf} = \langle \bar{\psi}_{cf} \gamma^0 \psi_{cf} \rangle = \frac{\hat{B}}{4\pi} \]

electron density

\[ \rho_{el} = -\frac{b}{4\pi} \]

\[ 2\nu_{el} = -\frac{1}{2\nu_{cf}} \]
Introduction/Motivation

Variants of this duality has been proved/studied in the context of (3+1D) bulk-(2+1 D) boundary correspondence (C. Wang, T. Senthil; M. Metlitski, A. Vishwanath; M. Metlitski, 2016), but it would also be nice if one can show this duality in a dynamical way that is intrinsically 2+1 D.

There exists a dynamical duality that is similar in nature, called “mirror duality”, which is supersymmetric. The goal of this talk is to introduce this duality, and demonstrate that despite being supersymmetric, the response to external magnetic field is dominated by the “fermionic” part in the low energy dynamics analogous to that of Son’s duality, therefore making it a useful tool to study dirac fermions near half-filling.

Son’s duality (conjectured)

Mirror duality (dynamical)
Introducing Mirror Duality

Mirror Duality: (K. Intriligator and N. Seiberg 96’)

\[ \mathcal{N} = 4 \text{ SUSY } U(1) \text{ gauge theory with one flavor of charged multiplets (Theory B)} \]

\[ L = -\frac{1}{4g^2} f_{\mu\nu}^2 + i\bar{\psi} \gamma^\mu (\partial_\mu + ia_\mu) \psi + \text{ SUSY partners…} \]

\[ \mathcal{N} = 4 \text{ SUSY free multiplet (Theory A)} \]

\[ L = i\bar{\Psi} \gamma^\mu \partial_\mu \Psi + \text{ SUSY partners…} \]

In anticipation to connect with Son’s duality, we identify the theory with dynamical gauge field (Theory B) as the composite fermion theory; and the free theory (Theory A) as the one containing physical electrons.
Introducing Mirror Duality

What are the “$ (\mathcal{N} = 4) $ SUSY partners”?

SUSY: doubling of degrees of freedoms  

\[
\begin{align*}
\text{boson} & \quad \longleftrightarrow \quad \text{fermion} \\
\text{Theory B} & \\
\end{align*}
\]

\[
\begin{align*}
& a_\mu \quad \rightarrow \quad \mathcal{N} = 4 \text{ vector multiplet } \mathcal{V} : = \{ a_\mu, \phi, \lambda_{ia} \} \\
& \psi \quad \rightarrow \quad \mathcal{N} = 4 \text{ hypermultiplet } \mathcal{Q} : = \{(u^+, \psi^+), (u^-, \psi^-)\}
\end{align*}
\]

\[
L = -\frac{1}{4g^2} f_{\mu\nu}^2 + i\bar{\psi}\gamma^\mu (\partial_\mu + ia_\mu) \psi + \text{ SUSY partners } + \text{ SUSY couplings compatible with } i\bar{\psi}a_\mu \gamma^\mu \psi
\]

SUSY extension

\[
\begin{align*}
\mathcal{L}^B(\mathcal{V}, \mathcal{Q}) &= \mathcal{L}(\mathcal{V}) + \mathcal{L}(\mathcal{Q}, \mathcal{V}) \\
\mathcal{L}(\mathcal{V}) &= \frac{1}{g^2} \left( -\frac{1}{4} f_{\mu\nu}^2 + \frac{1}{2} \left( \partial_\phi \right)^2 + i\bar{\lambda} \phi \lambda \right) \\
\mathcal{L}(\mathcal{Q}, \mathcal{V}) &= |Du^a|^2 + i\bar{\psi}^i D\psi^i - |\phi_{ij}|^2 |u^a|^2 - \phi_{ij} \bar{\psi}^i \psi^j \\
& \quad + \sqrt{2} \left( i\lambda_{ia} u^a \psi^i + \text{h.c.} \right) - \frac{g^2}{2} (u^a u^b)^2
\end{align*}
\]
Introducing Mirror Duality

What are the “(\(\mathcal{N} = 4\)) SUSY partners”?

SUSY: doubling of degrees of freedoms  

\[
\Psi_{el} \rightarrow \mathcal{N} = 4 \text{ hypermultiplet: } \{(v^+, \Psi^+), (v^-, \Psi^-)\}
\]

\[
L = \bar{\Psi}_{el} \gamma^\mu \partial_\mu \Psi_{el}
\]

SUSY extension

\[
L^A = \sum_{\pm} \left( |\partial v^\pm|^2 + i \bar{\Psi}^\pm \partial \Psi^\pm \right)
\]

An important component of Son’s duality is the way external electromagnetic sources are coupled across the duality. To connect it with mirror duality, we need to introduce the physical electromagnetic \(U(1)_{em}\) for the mirror pair.
Introducing Mirror Duality

There are various global symmetries on both sides of the duality. The power of mirror duality allows us to identify these global symmetries across.

(Theory B) topological symmetry $U(1)_J : J_{\mu}^{\text{top}} = \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} f^{\nu\rho}$

The topological symmetry acts in Theory B by shifting the “dual photon”: $\gamma \rightarrow \gamma + \epsilon$

defined by $f^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} \partial_{\rho} \gamma$, therefore only the (non-perturbative) dynamical vortices $e^{\frac{2\pi i}{\sigma} \gamma}$ are charged under the topological symmetry.

Identified by mirror duality

(Theory A) phase rotation $U(1)_A : (\nu^{\pm}, \Psi^{\pm}) \rightarrow e^{\pm i\theta} (\nu^{\pm}, \Psi^{\pm})$

In particular, the charges are carried by vortices in one theory, but by fundamental particles in the other. This demonstrates an important aspect of mirror duality: particle/vortex duality, augmented by super-partners.
Introducing Mirror Duality

We therefore identify the physical $U(1)_{em} = U(1)_J = U(1)_A$

Coupling to external source $A_\mu$:

$\mathcal{L}^B(V, Q, A_\mu) = L(V) + L(Q, V) + \frac{1}{4\pi} \epsilon^{\mu\nu\rho} f_{\mu\nu} A_\rho$

Theory B

$\mathcal{L}^A(A_\mu) = \sum_{\pm} \{(\partial_\mu \pm iA_\mu)\psi_{\pm}|^2 + \Psi_{\pm}\gamma^\mu (i\partial_\mu \mp A_\mu) \Psi_{\pm}\}$

Theory A

Reminder (Son’s conjecture):

$L_{el} = i\Psi_{el}\gamma^\mu (\partial_\mu + iA_\mu) \Psi_{el}$

$L_{CF} = i\psi_{cf}\gamma^\mu (\partial_\mu + 2ia_\mu) \psi_{cf} + \frac{1}{2\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda$

In conclusion, with the appropriate identification of $U(1)_{em}$, mirror duality becomes a SUSY variation of Son’s conjectured duality.
Introducing Mirror Duality

A very quick glimpse of why mirror duality works (S. Sachdev, X. Yin 2008):

Start from theory B (gauge theory), one can integrate out the charged matter to obtain the effective action for the gauge fields (vector multiplet). Due to the large number of SUSY present, the effective action is one-loop exact, and is described explicitly by a non-linear sigma model on the coulomb branch:

\[
\mathcal{L}_{\text{eff}} = \frac{1}{g^2} \left( H(\phi) \left( \partial_{\mu} \phi \right)^2 + H^{-1}(\phi) \left( \partial_{\mu} \gamma + \frac{1}{2\pi} \vec{\omega}(\phi) \cdot \partial_{\mu} \phi \right)^2 \right) + \text{fermionic part}
\]

\[
H(\phi) = 1 + \frac{g^2}{4\pi|\phi|}
\]

\[
\vec{\nabla} \times \vec{\omega}(\phi) = \vec{\nabla} H(\phi)
\]

low energy regime corresponds to the limit \( g \to \infty \), in which we can find an explicit change of variables that brings the effective action into a free theory form.
Introducing Mirror Duality

A very quick glimpse of why mirror duality works (S. Sachdev, X. Yin 2008):

Change of variables:

\[ v_i = \begin{pmatrix} v_+ \\ v_- \end{pmatrix} = \sqrt{\frac{\mid \phi \mid}{2\pi}} e^{2\pi i\gamma/g^2} \begin{pmatrix} \cos \theta / 2 \\ e^{i\varphi} \sin \theta / 2 \end{pmatrix} \]

\[ \Psi_a = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} = \frac{1}{\sqrt{2|\phi|}} \lambda_{ai} v_i, \quad \vec{\phi} = |\vec{\phi}|(\cos \theta, \sin \theta \cos \varphi, \sin \theta \sin \varphi) \]

\[ \mathcal{L}_{\text{eff}} = \frac{1}{g^2} \left( H(\phi) \left( \partial_\mu \vec{\phi} \right)^2 + H^{-1}(\phi) \left( \partial_\mu \gamma + \frac{1}{2\pi} \vec{\omega}(\phi) \cdot \partial_\mu \vec{\phi} \right)^2 \right) + \text{fermionic part} \]

change of variable

\[ \mathcal{L}_{\text{eff}} = \sum_i |\partial_\mu v_i|^2 + \sum_a \bar{\Psi}_a \gamma^\mu \partial_\mu \Psi_a + \mathcal{O}(|\vec{\phi}|/g^2) \]
Introducing Mirror Duality

A very quick glimpse of why mirror duality works

Change of variables:

\[ v_i = \begin{pmatrix} v_+ \\ v_- \end{pmatrix} = \sqrt{\frac{|\phi|}{2\pi}} e^{2\pi i \gamma/g^2} \begin{pmatrix} \cos \theta/2 \\ e^{i\varphi} \sin \theta/2 \end{pmatrix} \]

\[ \Psi_a = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix} = \frac{1}{\sqrt{2|\phi|}} \lambda_{ai} v_i, \quad \phi = |\phi| (\cos \theta, \sin \theta \cos \varphi, \sin \theta \sin \varphi) \]

\[ L_{\text{eff}} = \frac{1}{g^2} \left( H(\phi) \left( \partial_\mu \phi \right)^2 + H^{-1}(\phi) \left( \partial_\mu \gamma + \frac{1}{2\pi} \bar{\omega}(\phi) \cdot \partial_\mu \phi \right)^2 \right) + \text{fermionic part} \]

change of variable

\[ g \to \infty \]

Theory A!
Summary of Mirror duality

So far, we have introduced the mirror duality, and shown that by identifying the topological $U(1)_J$ on the gauge theory side as the physical $U(1)_{em}$, mirror duality becomes a SUSY version of Son’s conjecture.

Mirror duality can be verified by a change of variables that is explicitly particle/vortex duality, dressed by SUSY-partners.

Comparison:

<table>
<thead>
<tr>
<th>CF theory</th>
<th>$U(1)_{em}$</th>
<th>$U(1)_{gauge}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{cf}$</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$2\pi$ vortex</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>theory B</th>
<th>$U(1)_{em}$</th>
<th>$U(1)_{gauge}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_+, u_+$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\psi_-, u_-$</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$2\pi$ vortex</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{\phi}, \lambda$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Son’s conjecture

<table>
<thead>
<tr>
<th>el theory</th>
<th>$U(1)_{em}$</th>
<th>$U(1)_{gauge}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\Psi}_{el}$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>theory A</th>
<th>$U(1)_{em}$</th>
<th>$U(1)_{gauge}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi_+, v_+$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\Psi_-, v_-$</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>

$\Psi_\pm = \{2\pi$ vortex, $\bar{\phi}\}$

Mirror duality

$\Psi_\pm = \{2\pi$ vortex, $\bar{\phi}, \lambda\}$
Breaking SUSY — duality at half-filling

Although we have established the formal similarity between Son’s conjecture and mirror duality, due to the complicated interplay among the SUSY partners, the low energy dynamics upon deforming with external EM source may be completely different from Son’s result (i.e. Half-filling —> CF fermi-surface).

The goal of the this section is to argue that by deforming mirror duality with external magnetic field, the bosonic SUSY partners decouple at low energies on both sides, the effective descriptions are therefore driven by the fermionic responses. We derive the low energy theories on both sides, by identifying them we therefore provide a dynamical realization of the composite fermi-surface(s) picture for half-filled Dirac fermions.
Breaking SUSY — duality at half-filling

Concern: does the deformation, which breaks SUSY, invalidate the duality, which relies on SUSY?

The SUSY duality has an intrinsic scale set by the dimensionful bare gauge coupling $g$. By turning on deformations of scale $M$, SUSY-breaking corrections are controlled by $\mathcal{O}(M/g^2)$.

Theory A = Free theory $+ \mathcal{O}(E/g^2)$
Concern: does the deformation, which breaks SUSY, invalidate the duality, which relies on SUSY?

The SUSY duality has an intrinsic scale set by the dimensionful bare gauge coupling $g$. By turning on deformations of scale $M$, SUSY-breaking corrections are controlled by $\mathcal{O}(M/g^2)$.

Theory A $\rightarrow$ Free theory $+ \mathcal{O}(E/g^2)$

$E/g^2 \rightarrow 0$
Breaking SUSY — duality at half-filling

Concern: does the deformation, which breaks SUSY, invalidate the duality, which relies on SUSY?

The SUSY duality has an intrinsic scale set by the dimensionful bare gauge coupling $g$. By turning on deformations of scale $M$, SUSY-breaking corrections are controlled by $O(M/g^2)$.

\[
\text{SUSY-breaking sources } \sim M
\]

Theory A $\rightarrow$ Free theory $+ O(E/g^2)$  \[ E/g^2 \rightarrow 0 \]  Theory A $\rightarrow$ Free theory $+ O(M/g^2)$
Concern: does the deformation, which breaks SUSY, invalidate the duality, which relies on SUSY?

The SUSY duality has an intrinsic scale set by the dimensionful bare gauge coupling $g$. By turning on deformations of scale $M$, SUSY-breaking corrections are controlled by $O(M/g^2)$.

\[ \text{SUSY-breaking sources} \sim M \]

Theory A = Free theory + $O(E/g^2)$

\[ E/g^2 \to 0 \]

In particular, here we shall require that the background B field be weak:

\[ \sqrt{B} \ll g^2 \]
Breaking SUSY — duality at half-filling

Theory A: (free) charged Dirac fermions + relativistic bosons in constant background magnetic field $B$

spectrum: relativistic Landau levels

$$E_n^{\text{fermion}} = \pm \sqrt{2nB}$$
$$E_n^{\text{boson}} = \pm \sqrt{(2n + 1)B}$$

Project down to low energies: $E \ll \sqrt{B} \ll g^2$, scalars are gapped out.

Theory A: two copies of LLL Dirac fermions at neutrality

$$2 \times \begin{array}{c} \text{Half-filled Landau Levels} \\
\end{array}$$
Breaking SUSY — duality at half-filling

Theory B: \[ \mathcal{L}^B = L(\mathcal{V}) + L(Q, \mathcal{V}) - \frac{1}{2\pi} a_0 B \]

Project down to low energies: \( g \to \infty \), the kinetic term for the vector \( L(\mathcal{V}) \) multiplet is suppressed by a factor of \( 1/g^2 \).
Breaking SUSY — duality at half-filling

Theory B: \[ \mathcal{L}^B = I(V) + L(Q, V) - \frac{1}{2\pi} a_0 B \]

How do the SUSY field content respond to the source \(-\frac{1}{2\pi} a_0 B\)?

Solving the system:

\[ \mathcal{L}^B = |D u^a|^2 + i \bar{\psi} \gamma^\mu \partial_\mu \psi^i - |\bar{\phi}|^2 |u^a|^2 - \phi_{ij} \bar{\psi}^i \psi^j + \sqrt{2} (i \lambda_{ia} u^a \psi^i + h.c.) - \frac{g^2}{2} (u^a u^b)^2 - \frac{1}{4\pi} a_0 B \]

Charged scalars \((u_+, u_-)\): quartic potential \(V(u) = \frac{g^2}{2} (u^a u^b)^2\)

as \(g \rightarrow \infty\), \(V(u)\) turns into a hard constraint: \(\langle u_+ \rangle = \langle u_- \rangle = 0\)

\(a_0\) Eqn of Motion: \(\langle \psi_+^\dagger \psi_+ \rangle - \langle \psi_-^\dagger \psi_- \rangle = -\frac{1}{2\pi} B \)

\[ \rightarrow \text{Total fermi surface } \frac{1}{2\pi} B, \text{ shared between } \psi_\pm \]
Breaking SUSY — duality at half-filling

Theory B:  
\[ \langle \psi_+^\dagger \psi_+ \rangle - \langle \psi_-^\dagger \psi_- \rangle = -\frac{1}{2\pi} B \]

How is the total fermi surface distributed between \( \psi_\pm \)?

There are other global symmetries that we can identify across the duality:

Theory B:  \( SU(2) \) rotation of the \( (\psi_+, \psi_*^-) \) doublet

Mirror duality

Theory A:  \( SU(2) \) rotation of the \( (v_+, v_*^-) \) doublet

\( SU(2) \) rotation symmetry should be respected by the two fermi-surfaces in Theory B:

\[ \rightarrow \langle \psi_+^\dagger \psi_+ \rangle = -\langle \psi_-^\dagger \psi_- \rangle \]

\( (v_+, v_*^-) \) massive due to laudau level.

\( SU(2) \) is not spontaneously broken
Breaking SUSY — duality at half-filling

Theory B: \[
\langle \psi_+^\dagger \psi_+ \rangle = -\langle \psi_-^\dagger \psi_- \rangle = -\frac{1}{4\pi} B
\]

Therefore, we deduced that the response to external magnetic field $B$ is the formation of two equal Fermi-surfaces of the Dirac fermions $(\psi_+, \psi_-)$, with Fermi momenta $k_F \sim \sqrt{B}$. In theory A, we projected out the scalars $(v^+, v^-)$ by going down to energies $E \ll \sqrt{B} \sim k_F$, i.e. the corresponding regime in theory B is the Fermi-liquid regime.

What survives in the low energy sector that couple to the Fermi-surfaces?
Breaking SUSY — duality at half-filling

Theory B: low energy effective theory near Fermi surfaces: \( E \ll k_F \)

\[
\mathcal{L}^B = |Du^a|^2 + i\bar{\psi}\gamma^\mu \partial_\mu \psi^j - |\phi|^2 |\bar{u}|^2 \\
-\phi_{ij}\bar{\psi}^i\psi^j + \sqrt{2} (i\lambda_i u^a \psi^i + h.c.) - \frac{g^2}{2} (u^a u^b)^2
\]
Breaking SUSY — duality at half-filling

Theory B: low energy effective theory near Fermi surfaces:  \( E \ll k_F \)

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\]

\( \{ \phi, u_{\pm} \} : \) without fermi-surfaces, quantum corrections to scalar masses cancel out between fermion loops and boson loops by supersymmetry; with the \( (\psi_+, \psi_-) \) fermi surfaces, there is a deficit of (negative) fermion loop contribution due to suppression of filled states \( |k| < k_F \).

\[
M_{\phi,u}^2 = \quad = 0 \quad \rightarrow \quad M_{\phi,u}^2 = \quad > 0
\]

Positive quantum correction to scalar masses: \( m_{\phi}^2 \sim m_u^2 \sim g^2 k_F \)
Breaking SUSY — duality at half-filling

Theory B: low energy effective theory near Fermi surfaces: \( E \ll k_F \)

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\mathcal{L}^B = |D u^a|^2 + i \bar{\psi} \gamma^\mu \partial_\mu \psi^j - |\phi|^2 |\bar{u}|^2
- \phi_{ij} \bar{\psi}^i \psi^j + \sqrt{2} (i \lambda_{ia} u^a \psi^i + h.c.) - \frac{g^2}{2} (u^a u^b)^2
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\( \left\{ \phi, u^\pm \right\} \): without fermi-surfaces, quantum corrections to scalar masses cancel out between fermion loops and boson loops by supersymmetry; with the \( (\psi_+, \psi_-) \) fermi surfaces, there is a deficit of (negative) fermion loop contribution due to suppression of filled states \( |k| < k_F \).

\[
M_{\phi,u}^2 = \begin{array}{ccc} \square & - & \square \end{array} = 0 \quad \rightarrow \quad M_{\phi,u}^2 = \begin{array}{ccc} \square & - & \square \end{array} > 0
\]

Positive quantum correction to scalar masses: \( m_\phi^2 \sim m_u^2 \sim g^2 k_F \)
Breaking SUSY — duality at half-filling

Theory B: low energy effective theory near Fermi surfaces: \( E \ll k_F \)

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\mathcal{L}^B = |Du^a|^2 + i\bar{\psi}\gamma^\mu \partial_\mu \psi^j - |\phi|^2 |\bar{u}|^2 \\
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\]

\[
\lambda_{ia} : \quad S_\lambda = i\sqrt{2} \int_{p,k} \lambda_{ia}(-k)u^a(-p)\psi^i(p+k) + h.c.
\]
Breaking SUSY — duality at half-filling

Theory B: low energy effective theory near Fermi surfaces: \( E \ll k_F \)

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\]

\[
\lambda_{ia} : \quad S_\lambda = i \sqrt{2} \int_{p,k} \lambda_{ia} (-k) u^a (-p) \psi^i (p + k) + h.c.
\]

\[
m_u^2 \sim g^2 k_F, \text{ integrate out}
\]

\[
\sim \frac{1}{g^2 k_F} \bar{\lambda} \lambda \bar{\psi} \psi
\]

Gaugino does not get massive, but it decouples from the Fermi surfaces, and becomes a “spectator” in the low energy sector.
In conclusion, for \( E \ll \sqrt{B} \ll g^2 \):

**Theory A:** \( 2 \times \) Half-filled Landau Levels under \( B \)

**Mirror duality**

**Theory B:** \( 2 \times \) composite fermi surface \( k_F \sim \sqrt{B} \)

+ emergent gauge field \( a_\mu \)
+ decoupled gaugino \( \lambda \)
To summarize, we have shown that:

Mirror duality, with the right identification of $U(1)_{\text{em}}$, is a supersymmetric variant of Son’s duality

Despite the SUSY spectrum, the response of the mirror pair to external magnetic field is dominated by the fermionic part, and we derived a double-copied version of the composite fermi surface — half-filled LL duality.

Effect of coulomb interactions: $z = 2$ scaling; no BCS instabilities; beta function predicts fixed points for the BCS couplings,
Conclusions/future directions

Some future directions:

- Study systematically the IR phase of Theory B (other orders, NFL, etc).
- Turn on electric charges to study the mirror dual of Jain’s sequence states
- Turn on additional deformations to obtain different limits of the duality.
- Study the implications other pairs of mirror symmetric theories.

Thank you!