Measurement-Only Topological Quantum Computation

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UIUC Workshop on Topological Phases of Matter
October 24, 2008

work done in collaboration with:
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arXiv:0802.0279 (PRL '08) and arXiv:0808.1933

Introduction

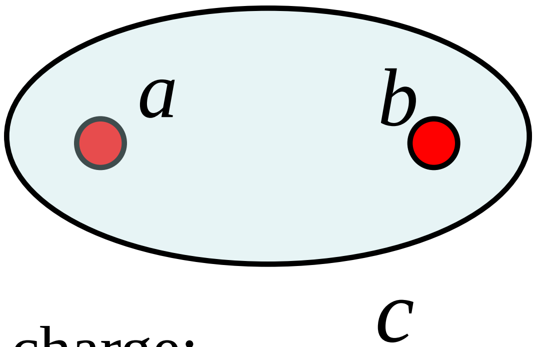
- Non-Abelian anyons are believed to exist in certain gapped two dimensional systems:

  - Fractional Quantum Hall Effect (ν=5/2, 12/5, …?)

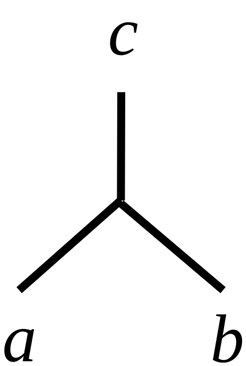
  - ruthenates, topological insulators, rapidly rotating bose condensates, quantum loop gases/string nets?
- If they exist, they could have application in quantum computation, providing naturally (“topologically protected”) fault-tolerant hardware.
- Assuming we have them at our disposal, what operations are necessary to implement topological quantum computation?

non-Abelian anyons

Localized topological charge:



Non-local collective topological charge:
(multiple values are possible)

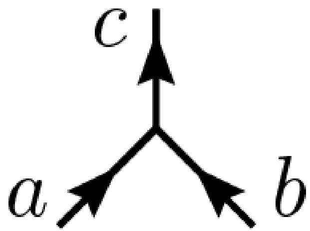


Fusion rules: $a \times b = \sum_c N_{ab}^c c$

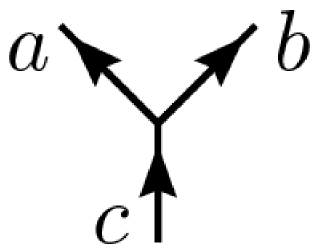
ang. mom. analogy: $\frac{1}{2} \times \frac{1}{2} = 0 + 1$

Hilbert space construct from state vectors associated with fusion/splitting channels of anyons.

Expressed diagrammatically:

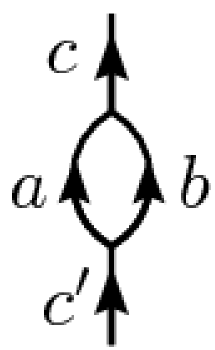


$$= \langle a, b; c| \in V_{ab}^{c}$$

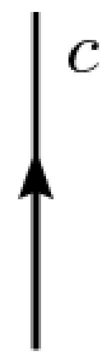


$$= |a, b; c\rangle \in V_{c}^{ab}$$

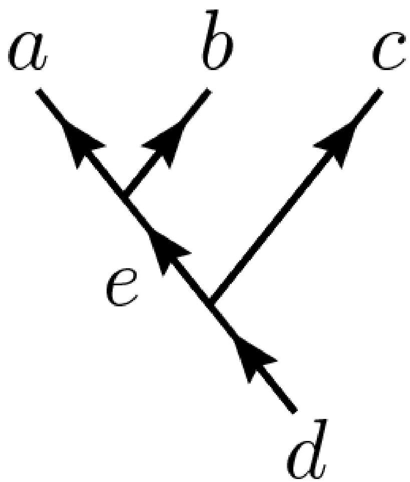
Inner product:



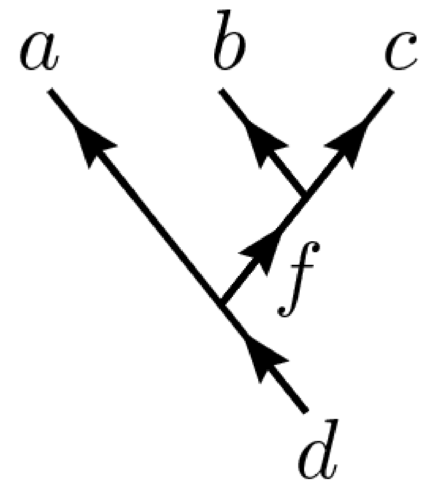
$$= \delta_{cc'}$$



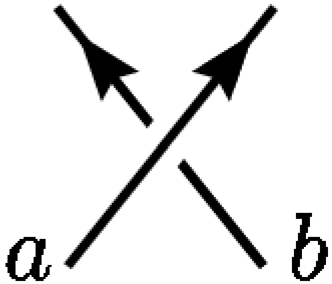
Associativity of fusing/splitting more than two anyons is specified by the unitary F-moves:

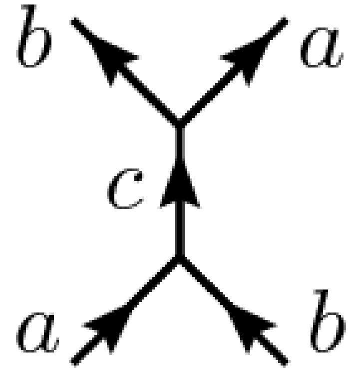


$$= \sum_f \left[F_d^{abc}\right]_{ef}$$



Braiding

$R^{ab} =$ 

$= \sum_c R_c^{ab}$ 

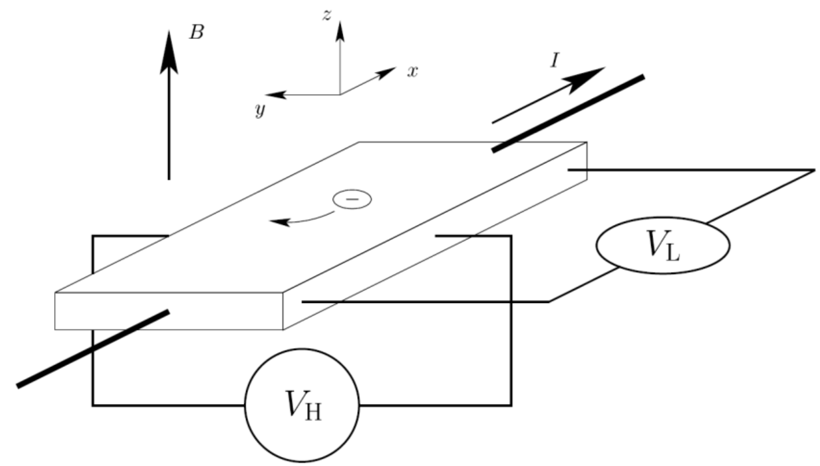
Can be non-Abelian if there are multiple fusion channels c

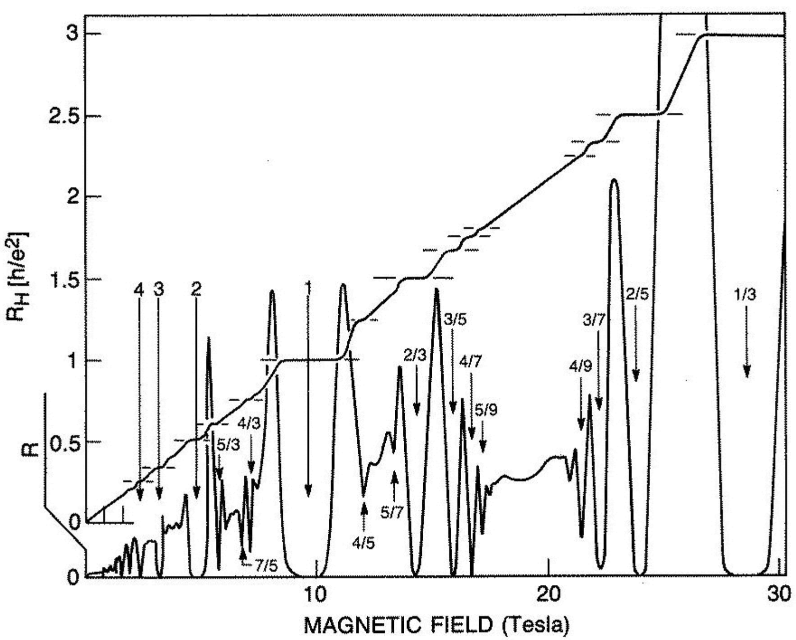
$$|\Psi_\alpha\rangle \mapsto U_{\alpha\beta}[R]|\Psi_\beta\rangle$$

Physical Anyons: Fractional Quantum Hall

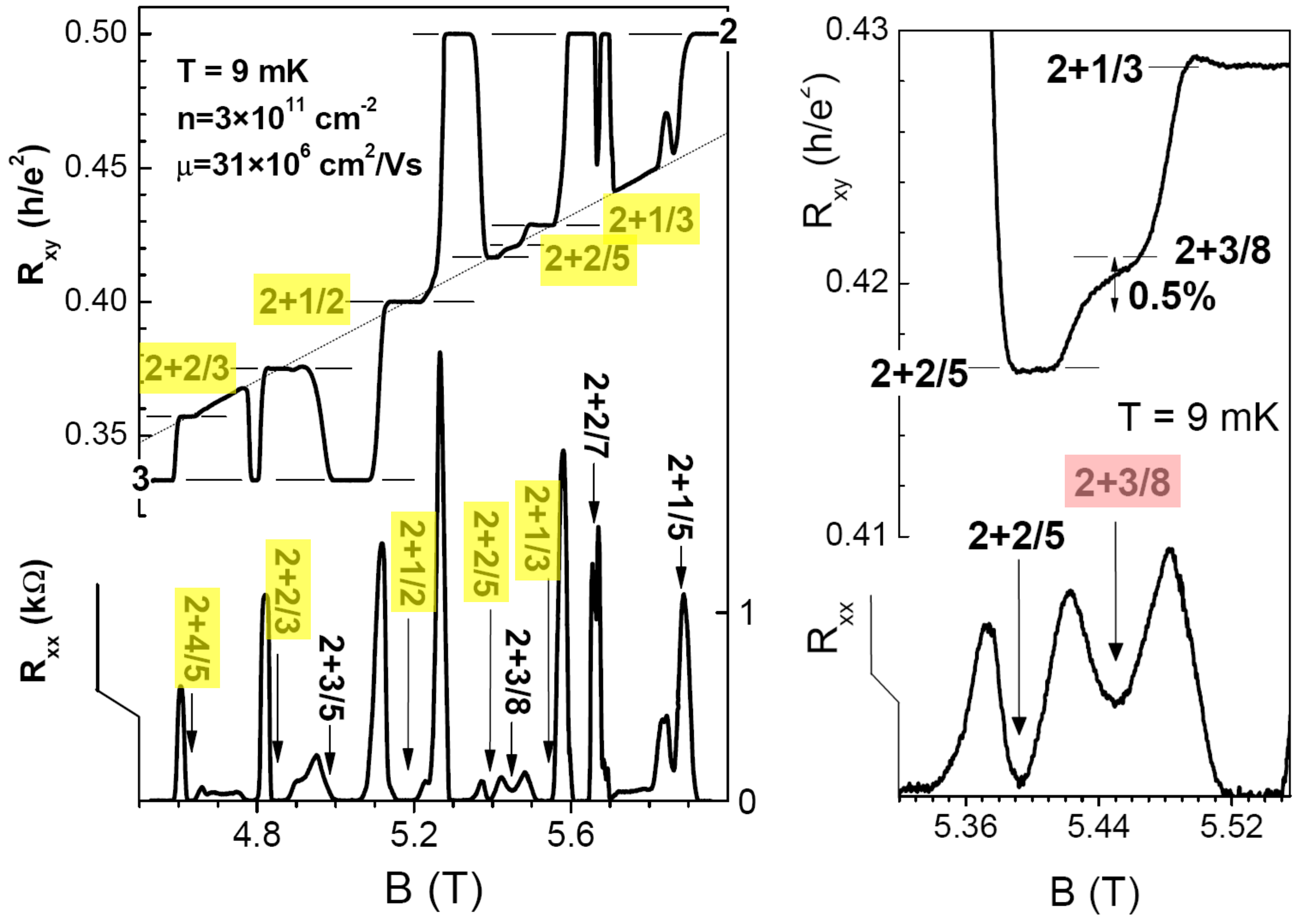
- 2DEG
- large B field (~ 10T)
- low temp (< 1K)
- gapped (incompressible)
- quantized filling fractions

$$\nu = \frac{n}{m}, \quad R_{xy} = \frac{1}{\nu}\frac{h}{e^2}, \quad R_{xx} = 0$$





- fractionally charged quasiparticles
- Abelian anyons at most filling fractions $\theta = \pi \frac{p}{m}$
- non-Abelian anyons in 2^{nd} Landau level, e.g. ν= 5/2, 12/5, …

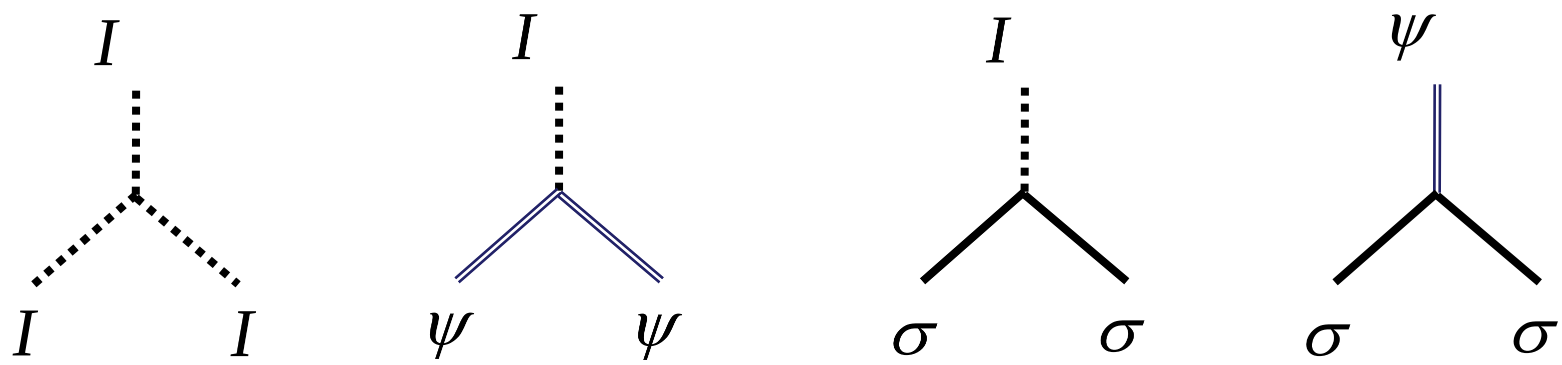

T = 9 mK
n=3×10^11 cm^-2
μ=31×10^6 cm^2/Vs
R_xy (h/e^2)
0.50
0.45
0.40
0.35
2
3
2+1/3
2+2/5
2+1/2
2+2/3
R_xx (kΩ)
2+4/5
2+2/3
2+3/5
2+1/2
2+2/5
2+1/3
2+3/8
2+2/7
2+1/5
1
0
4.8
5.2
5.6
B (T)
0.43
0.42
0.41
R_xy (h/e^2)
2+1/3
2+3/8
0.5%
2+2/5
T = 9 mK
2+3/8
2+2/5
R_xx
5.36
5.44
5.52
B (T)

Ising anyons

- $\nu = \frac{5}{2}$ FQH (Moore-Read `91)
- $\nu = \frac{12}{5}$ and other 2LL FQH? (PB and Slingerland `07)
- Kitaev honeycomb, topological insulators, ruthenates?

Topological charge types: $I,\ \sigma,\ \psi$

Fusion rules:



$\psi \times \psi = I$ $\qquad$ $\sigma \times \sigma = I + \psi$

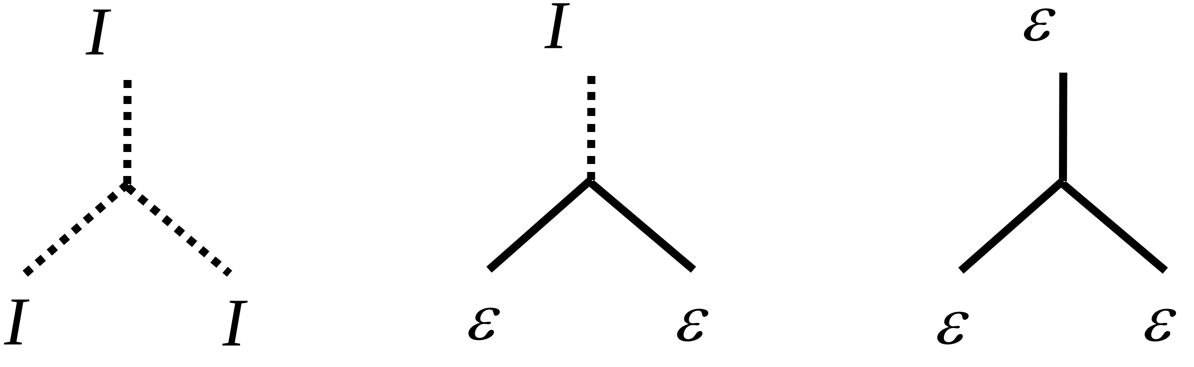
Fibonacci anyons

- $\nu = \frac{12}{5}$ FQH? (Read - Rezayi `98)

- string nets? (Levin - Wen `04, Fendley et. al. `08)

Particle types: $I, \ \varepsilon$

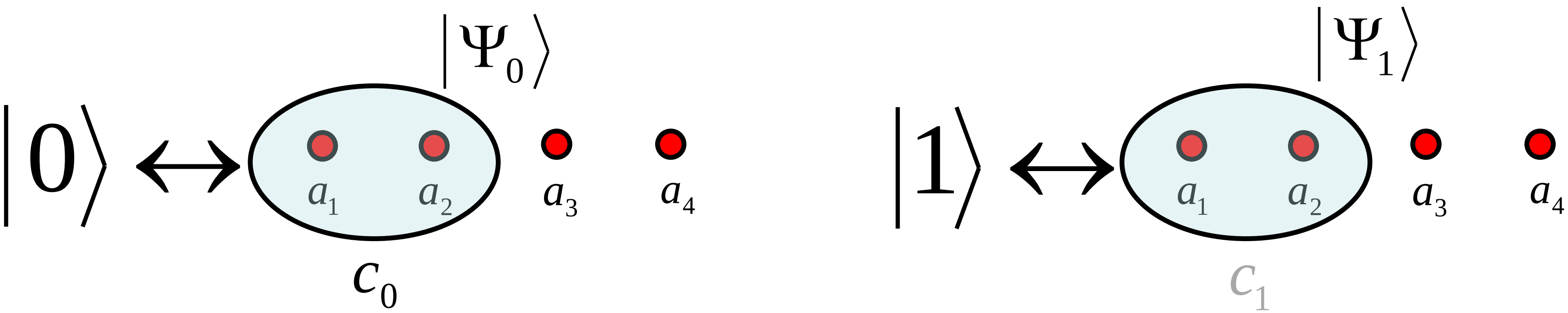
Fusion rules :



$$\varepsilon \times \varepsilon = I + \varepsilon$$

Topological Quantum Computation

(Kitaev, Preskill, Freedman, Larsen, Wang)



Topological Protection!

Ising: $a = \sigma, \; c_0 = I, \; c_1 = \psi$

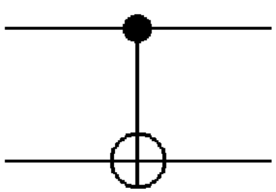
Fib: $a = \varepsilon, \; c_0 = I, \; c_1 = \varepsilon$

Topological Quantum Computation

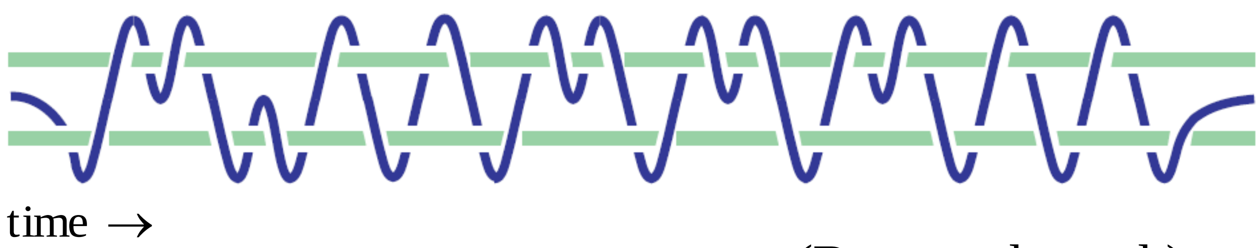
(Kitaev, Preskill, Freedman, Larsen, Wang)

$|\Psi_0\rangle$ $\qquad\qquad$ $|\Psi_1\rangle$

$|0\rangle \leftrightarrow$ $a_1 \quad a_2 \quad a_3 \quad a_4$ $\qquad$ $|1\rangle \leftrightarrow$ $a_1 \quad a_2 \quad a_3 \quad a_4$



$\leftrightarrow$



(Bonesteel, et. al.)

Is braiding computationally universal?

Ising: not quite

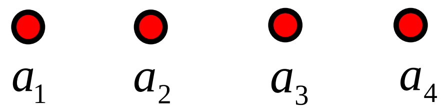
(must be supplemented)

Fib: yes!

Topological Quantum Computation

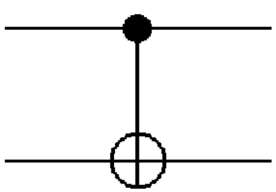
(Kitaev, Preskill, Freedman, Larsen, Wang)

$|\Psi_0\rangle$

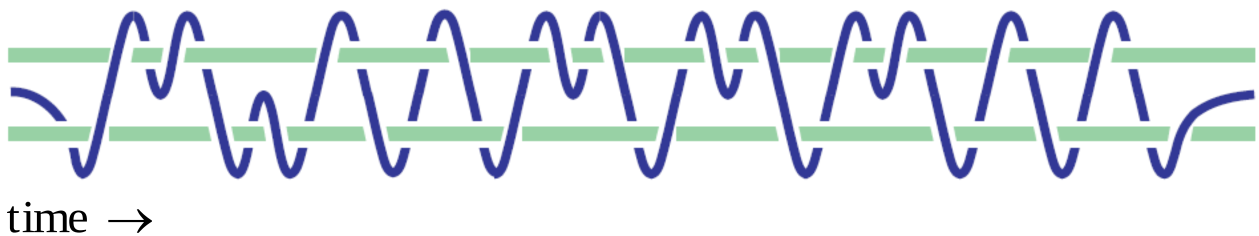
$|0\rangle \leftrightarrow$ 

$|\Psi_1\rangle$

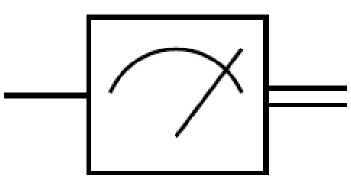
$|1\rangle \leftrightarrow$ 



$\leftrightarrow$



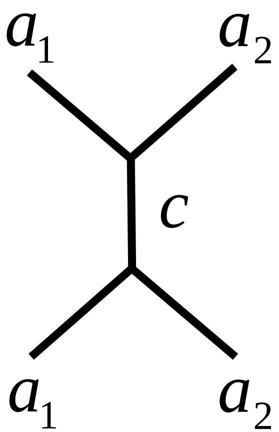
(Bonesteel, et. al.)



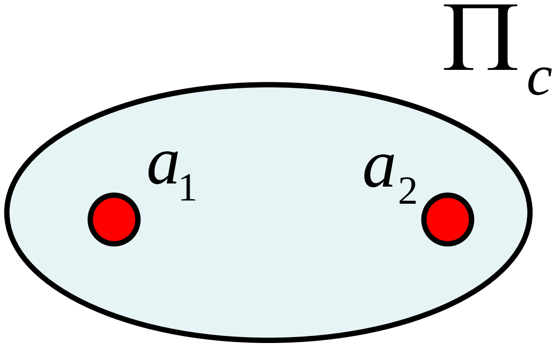
$\leftrightarrow$ Topological Charge Measurement

Topological Charge Measurement
(measures anyonic state)

$$\Pi_c = |a_1, a_2; c\rangle\langle a_1, a_2; c| =$$

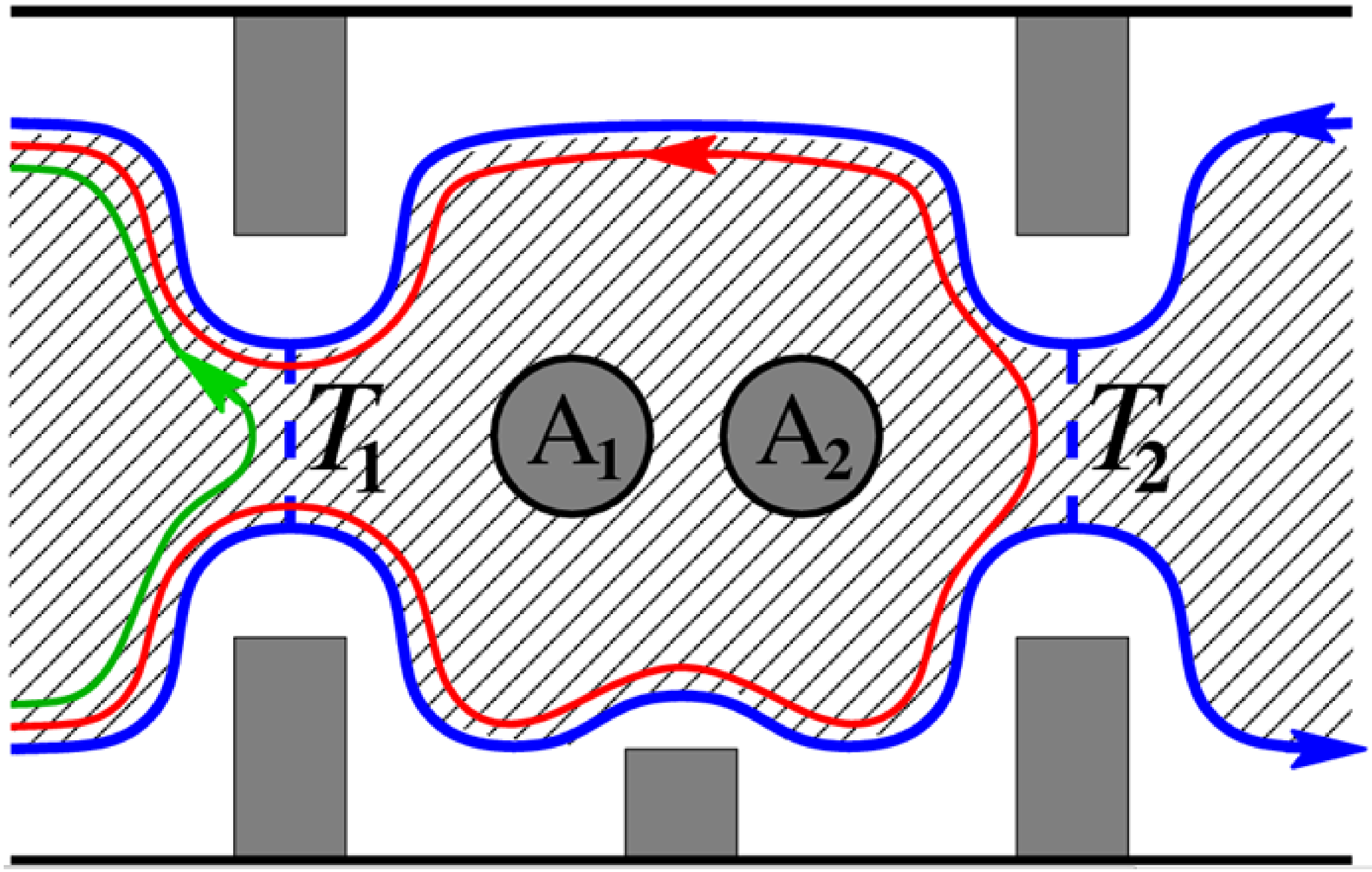


$$|\Psi\rangle \mapsto \frac{\Pi_c|\Psi\rangle}{\langle\Psi|\Pi_c|\Psi\rangle}$$

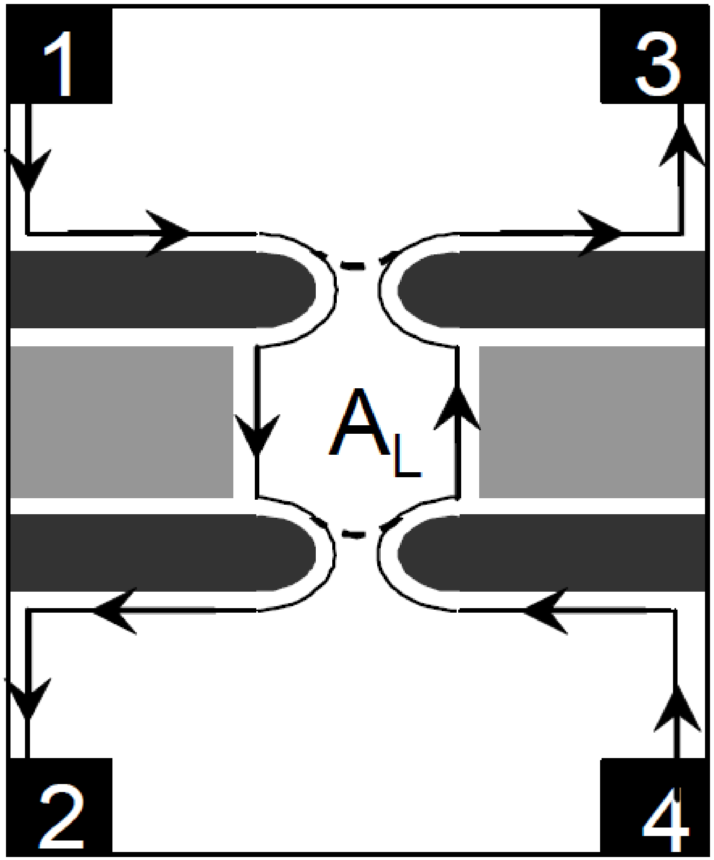


Topological Charge Measurement

e.g. FQH double point contact interferometer



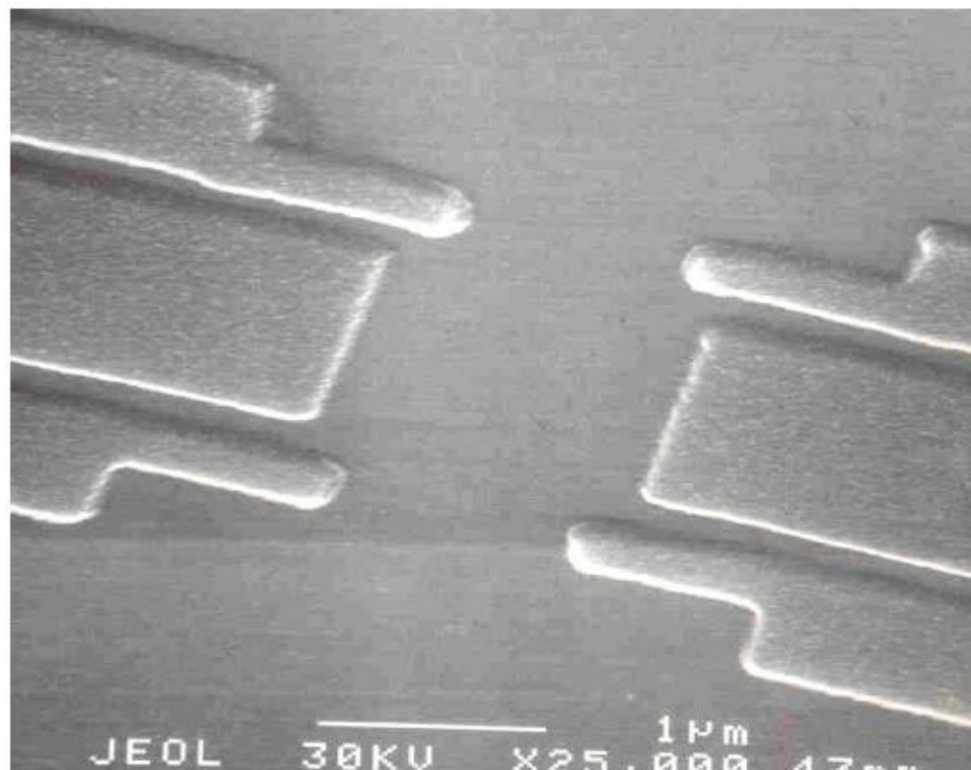
FQH interferometer



Willett, et. al.

for ν=5/2

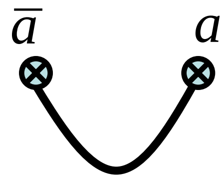
(also progress by: Marcus, Eisenstein, Kang, Heiblum, Goldman, etc.)



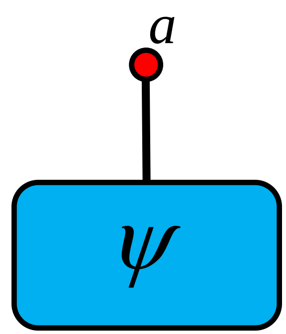
Anyonic State Teleportation

Entanglement Resource: maximally entangled anyon pair

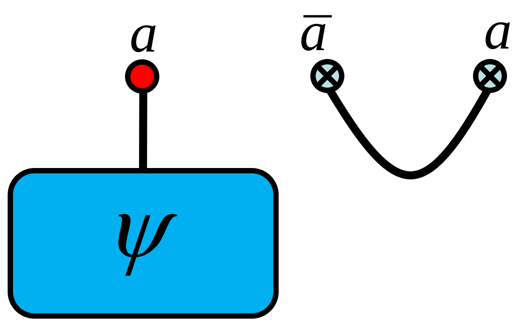
$$\left|\bar{a}, a; I\right\rangle =$$



Want to teleport: $\left|\psi\right\rangle =$



Form: $\left|\psi\right\rangle_1 \left|\bar{a}, a; I\right\rangle_{23} =$



and perform "Forced Measurement" on anyons

Anyonic State Teleportation

Forced Measurement
(projective)

$$\breve{\Pi}_{I}^{(12)}:$$

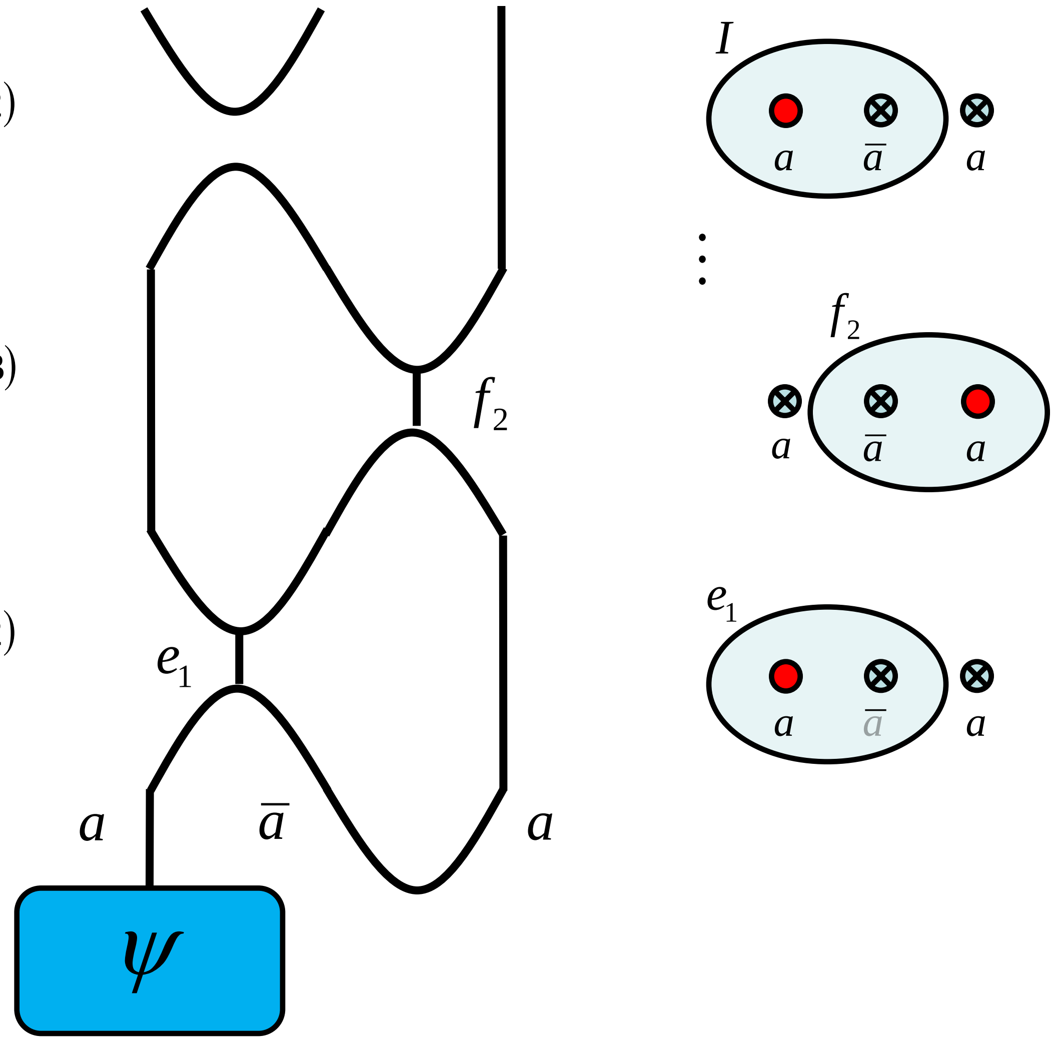
$$\Pi_{I}^{(12)}$$

$$\vdots$$

$$\Pi_{f_2}^{(23)}$$

$$\Pi_{e_1}^{(12)}$$

$$\left|\psi\right\rangle_{1}\left|\bar{a},a;I\right\rangle_{23}=$$

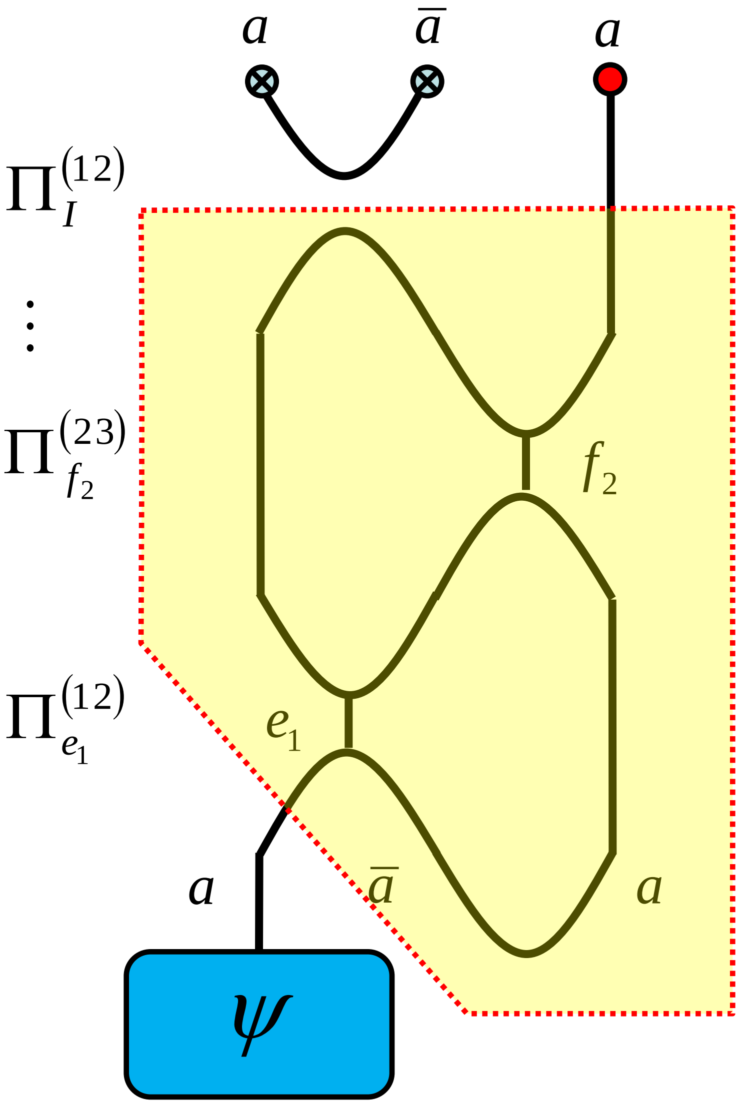


Anyonic State Teleportation

Forced Measurement

(projective)

$$\breve{\Pi}_{I}^{(12)}:$$



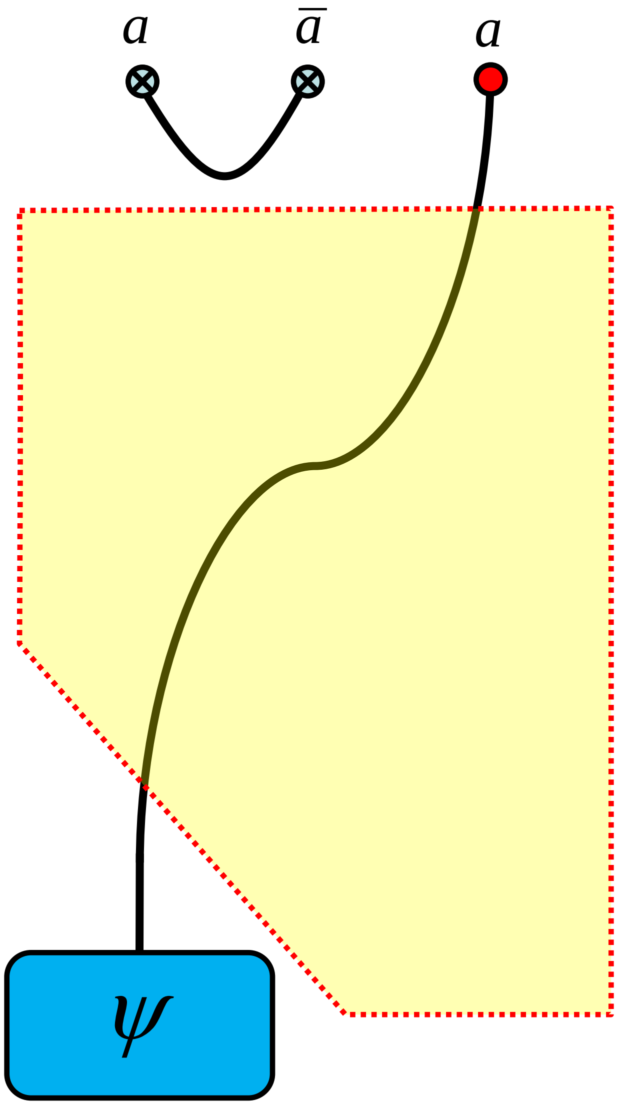
Anyonic State Teleportation

Forced
Measurement
(projective)

$$\breve{\Pi}_I^{(12)} \cong \Pi_I^{(12)}:$$

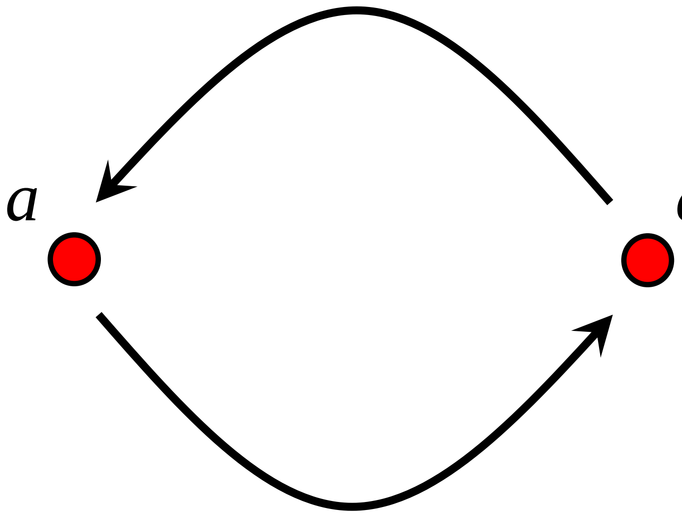
$$\left|\psi\right\rangle_1 \left|\bar{a},a;I\right\rangle_{23}$$

$$\mapsto \left|a,\bar{a};I\right\rangle_{12} \left|\psi\right\rangle_3 =$$

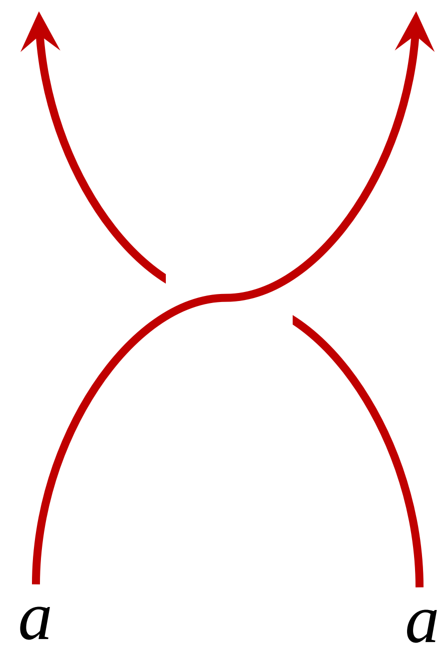


“Success” occurs with probability $\geq \frac{1}{d_a^2}$ for each repeat try.

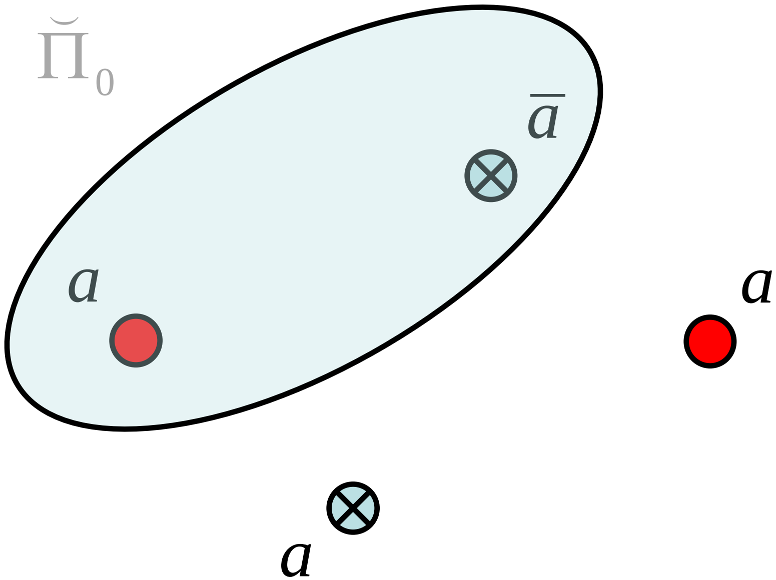
What good is this if we want to braid computational anyons?

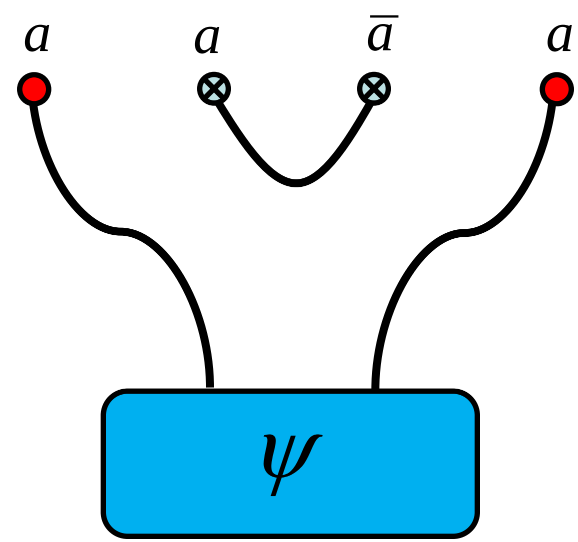


$R =$

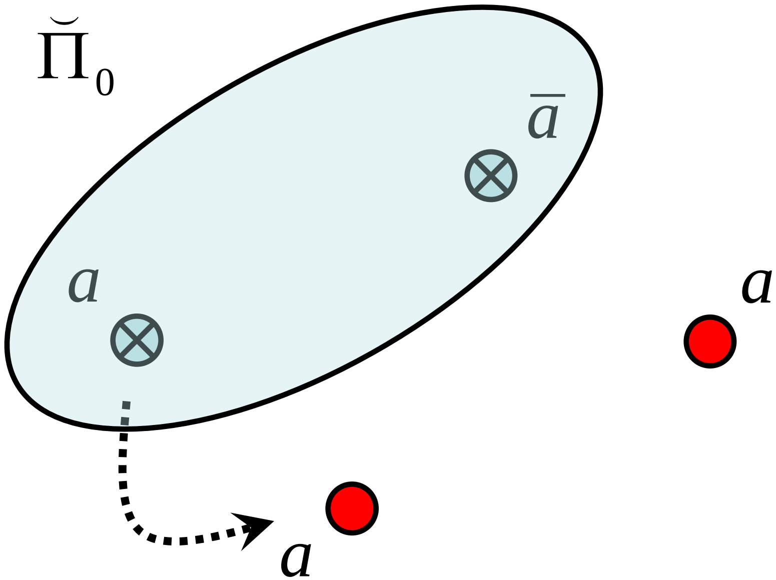


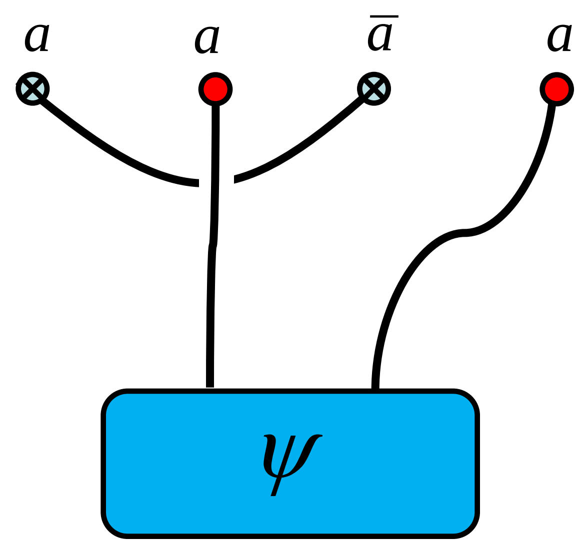
Use a maximally entangled pair and "forced measurements" for a series of teleportations



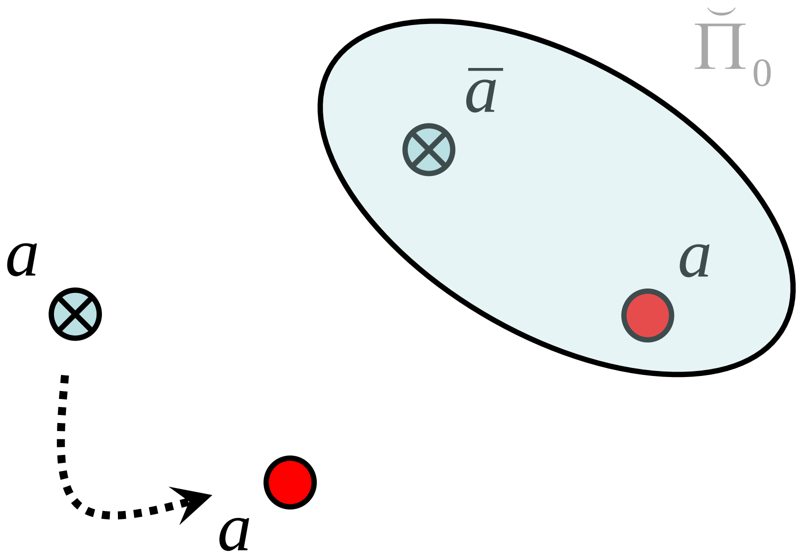


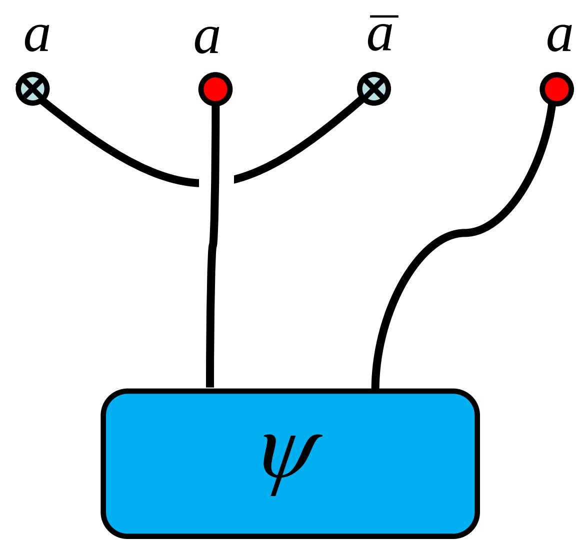
Use a maximally entangled pair and "forced measurements" for a series of teleportations



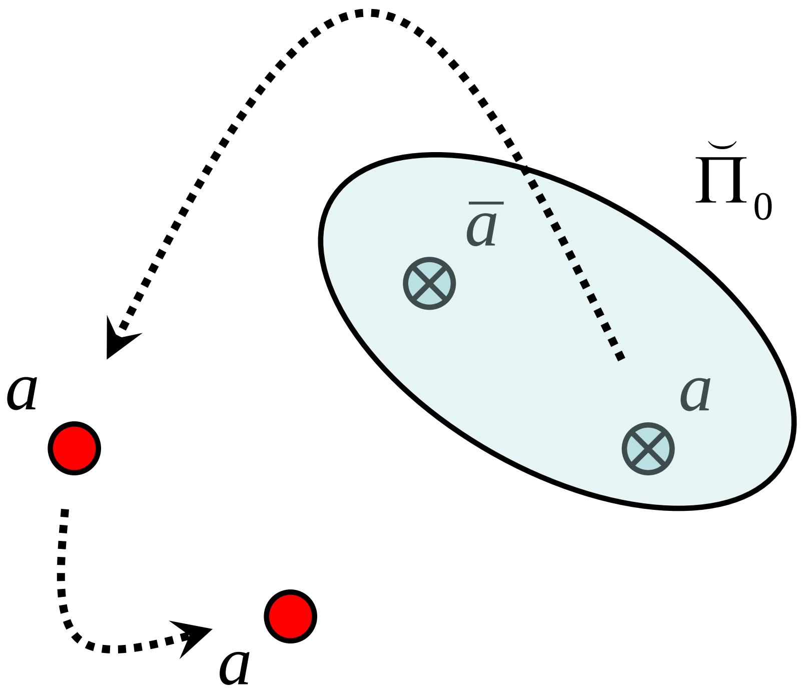


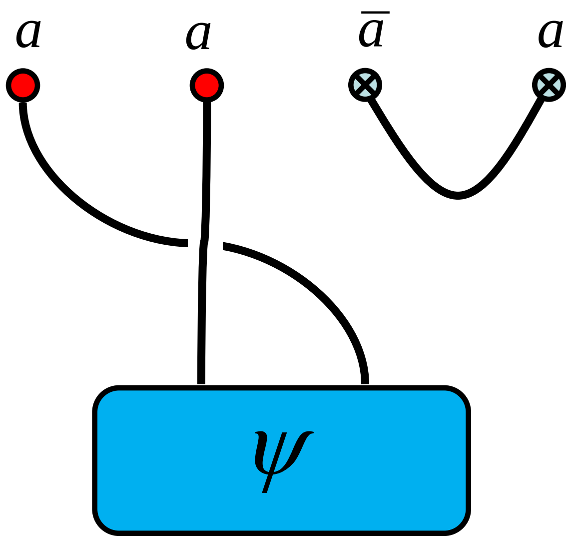
Use a maximally entangled pair and “forced measurements” for a series of teleportations



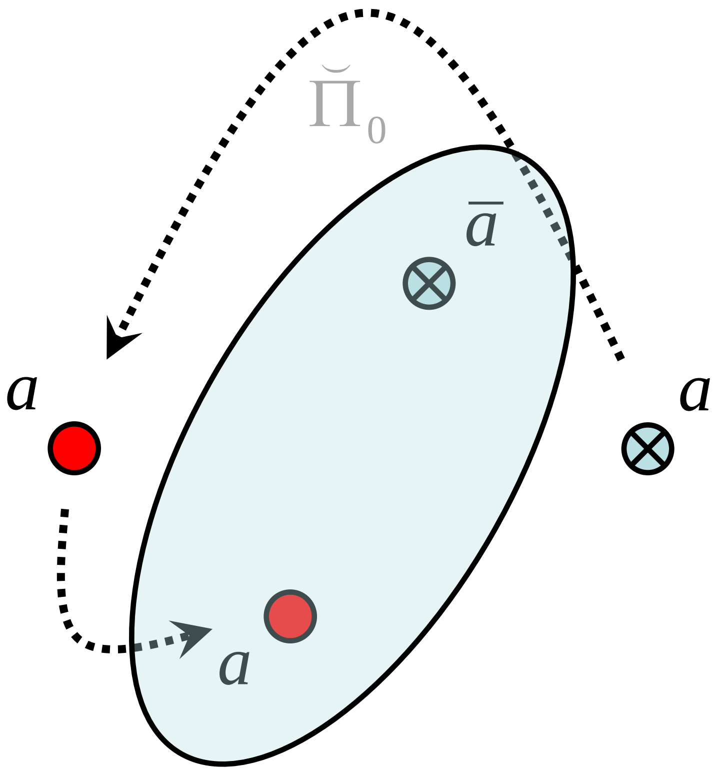


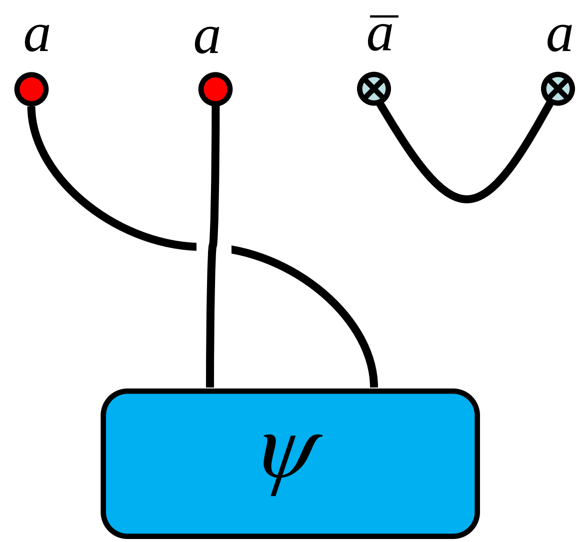
Use a maximally entangled pair and "forced measurements" for a series of teleportations



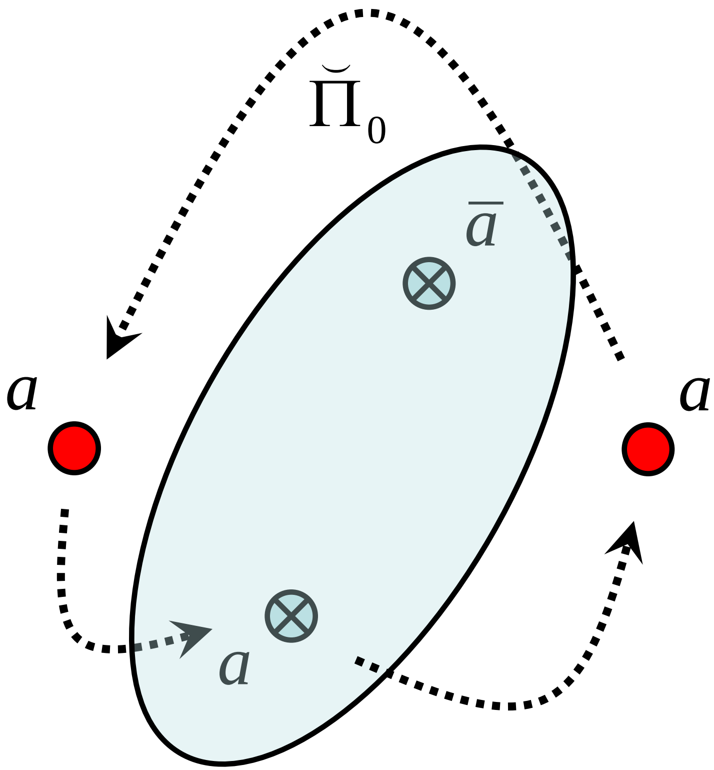


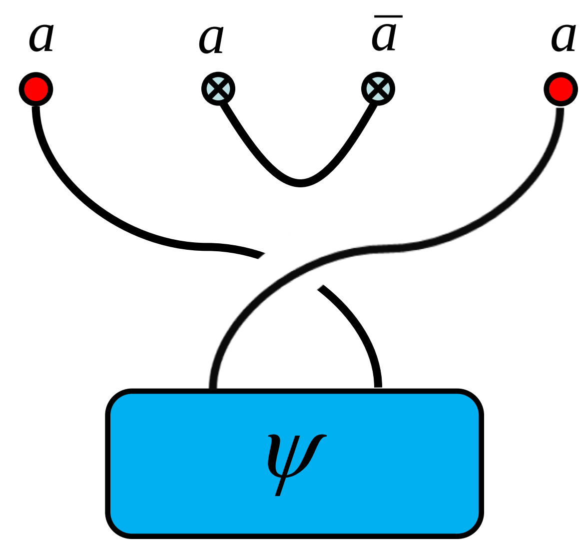
Use a maximally entangled pair and "forced measurements" for a series of teleportations



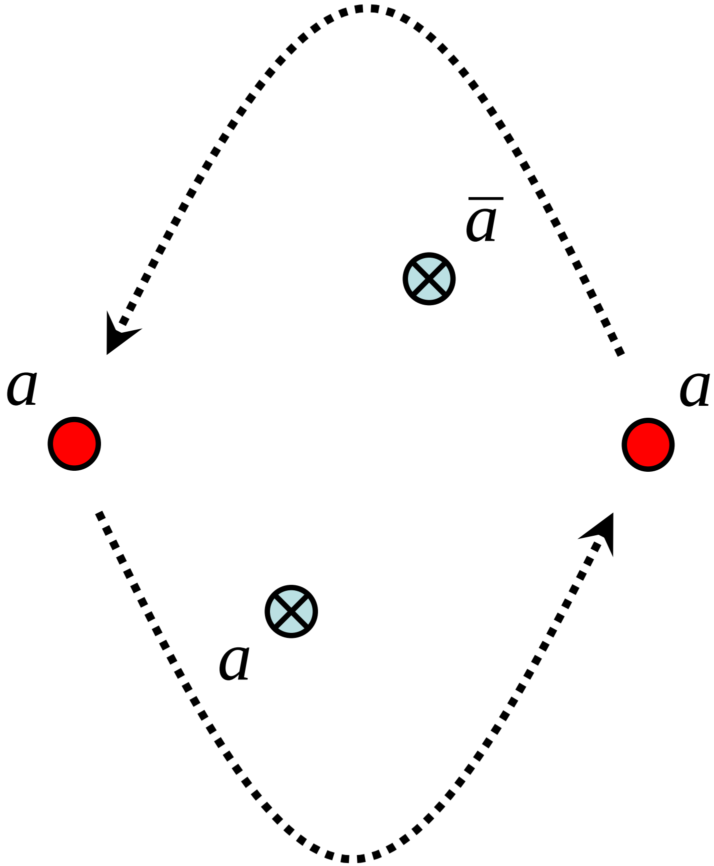


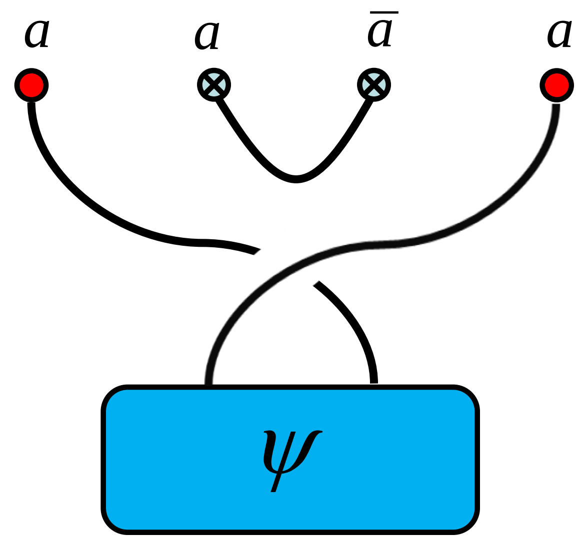
Use a maximally entangled pair and "forced measurements" for a series of teleportations





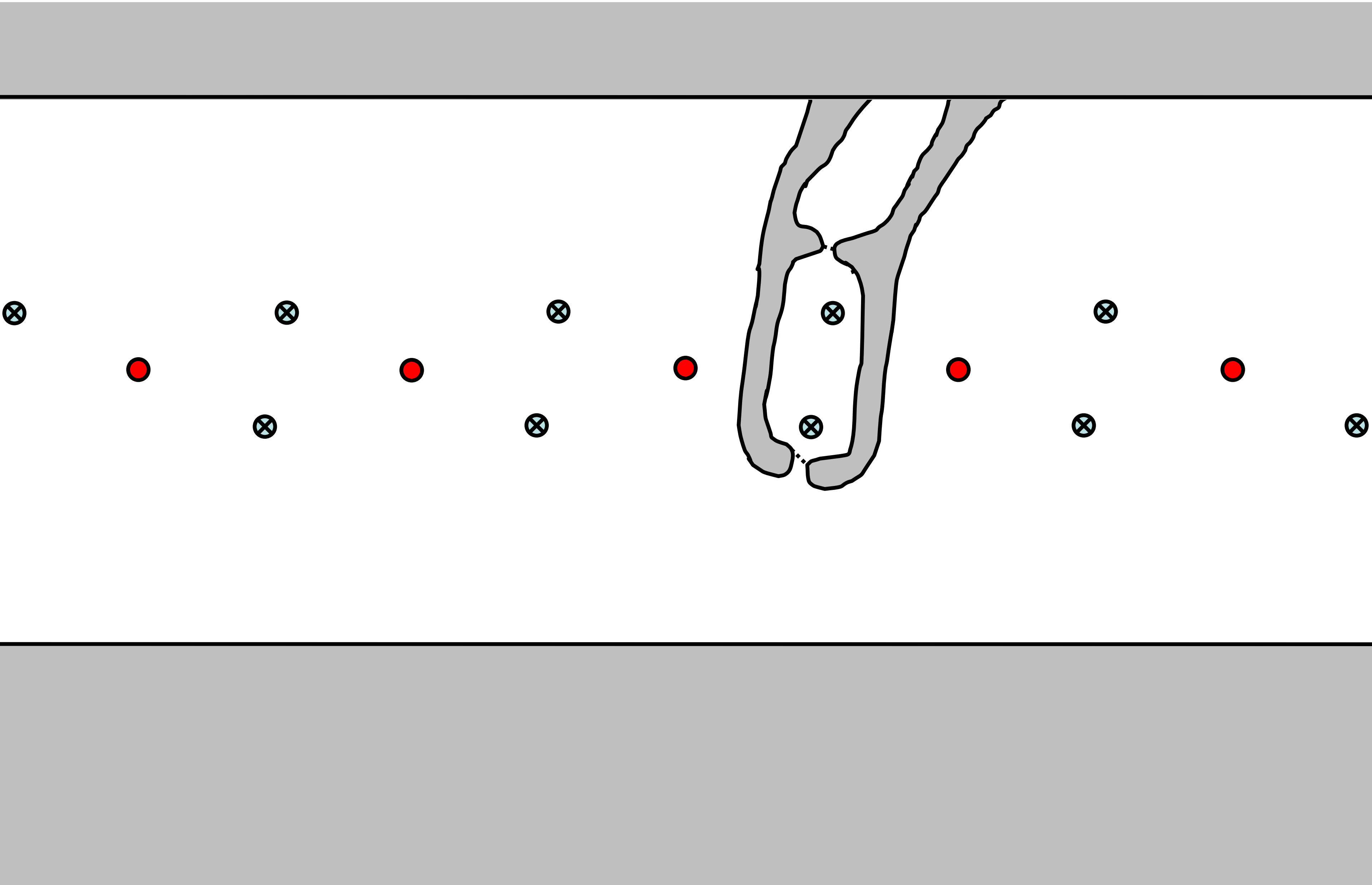
Measurement Simulated Braiding!



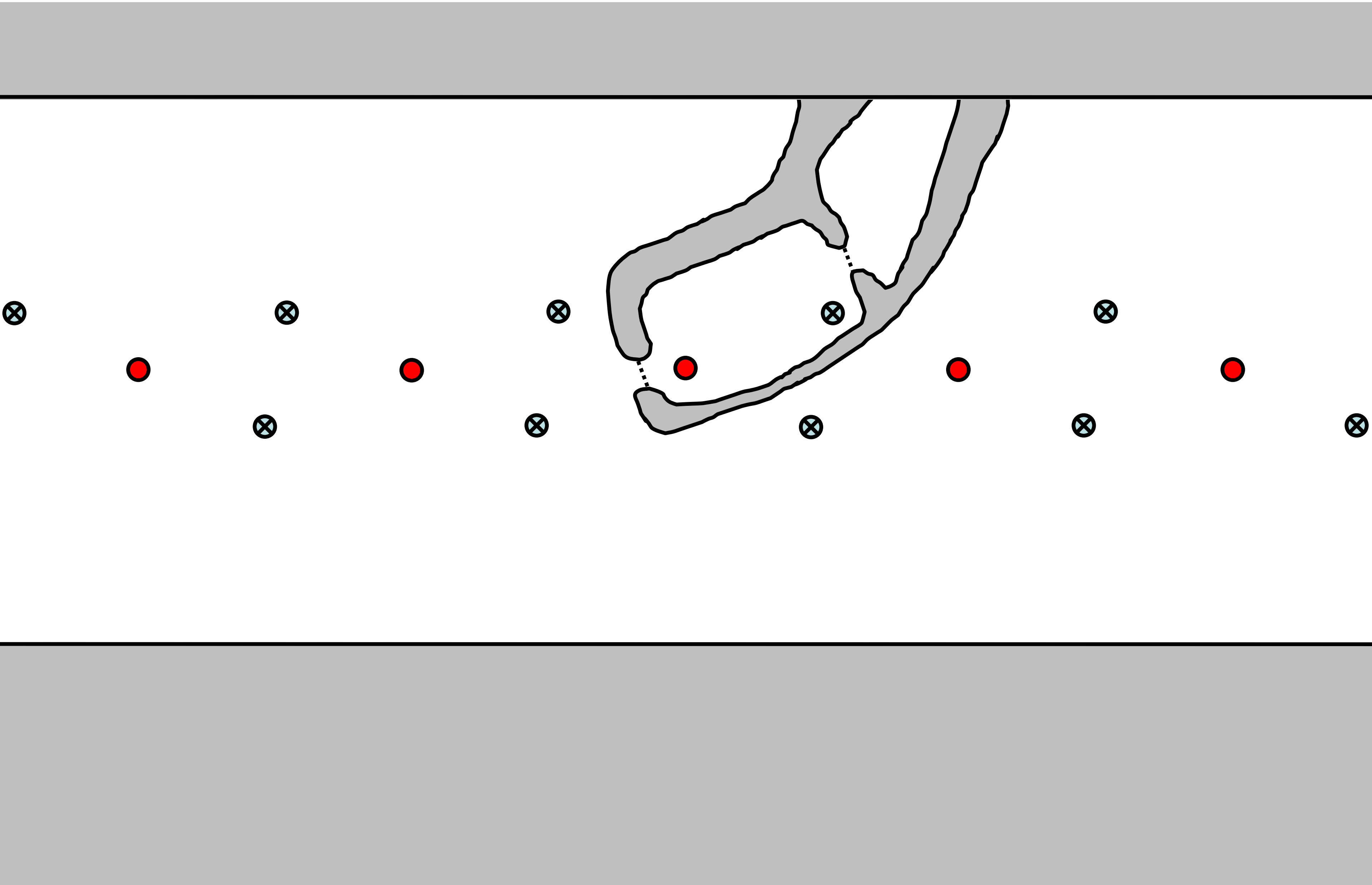


$$R^{(14)} \simeq \breve{\Pi}_0^{(23)} \breve{\Pi}_0^{(34)} \breve{\Pi}_0^{(13)}$$

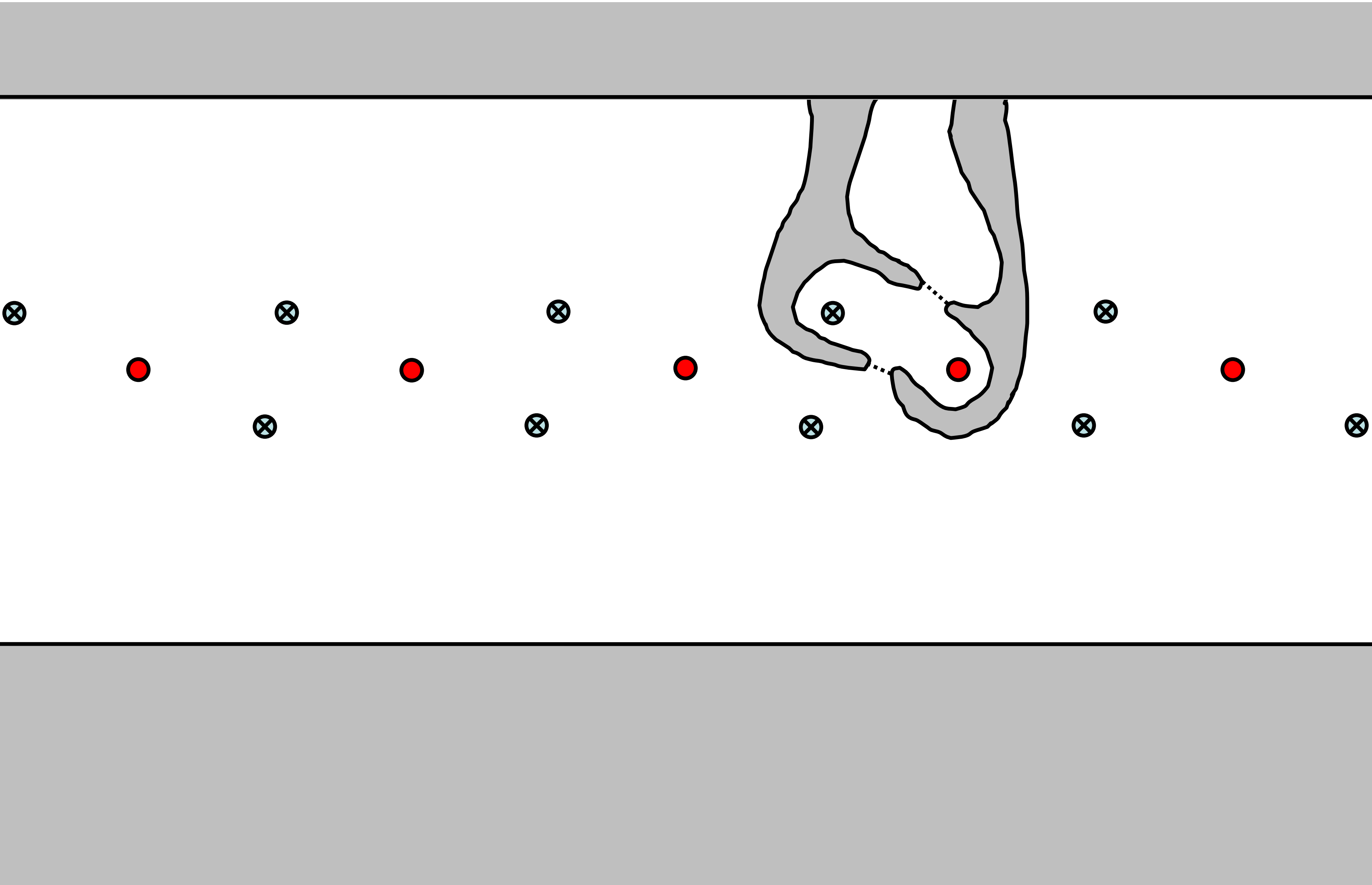
in FQH, for example



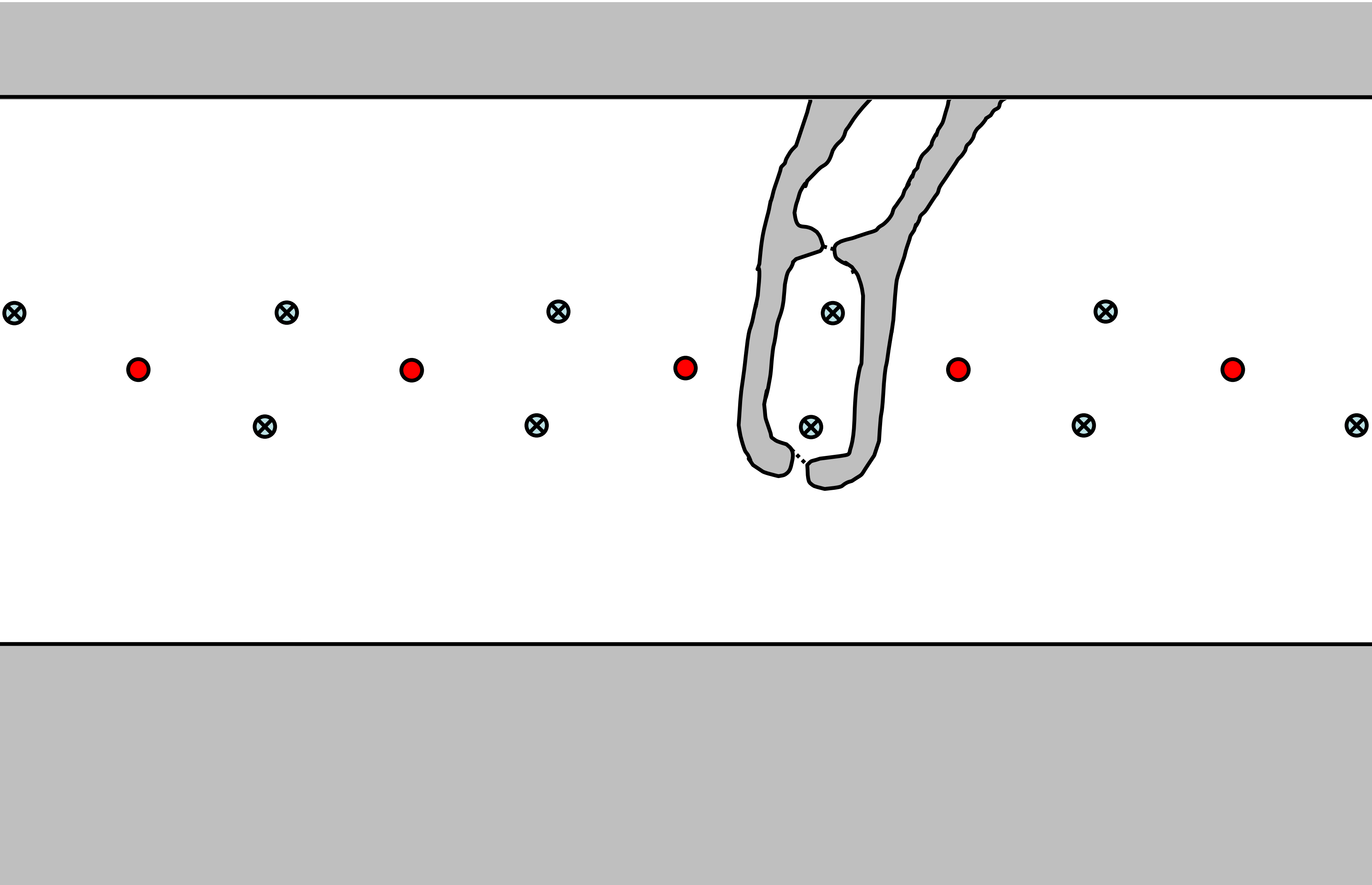
in FQH, for example



in FQH, for example



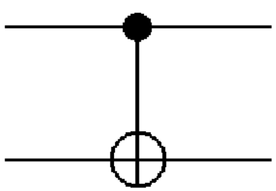
in FQH, for example



Measurement-Only Topological Quantum Computation

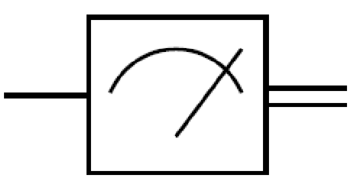
$$|0\rangle \leftrightarrow |\Psi_0\rangle \quad a_1 \quad a_2 \quad a_3 \quad a_4$$

$$|1\rangle \leftrightarrow |\Psi_1\rangle \quad a_1 \quad a_2 \quad a_3 \quad a_4$$



$\leftrightarrow$ Topological Charge Measurement

$\uparrow$ measurement simulated braiding



$\leftrightarrow$ Topological Charge Measurement

Measurement Generated Braiding!

Using Interferometric Measurements is similar but more complicated, requiring the density matrix description.
The resulting "forced measurement" procedure must include an additional measurement (of 8 or fewer anyons, i.e. still bounded size) in each teleportation attempt to ensure the overall charge of the topological qubits being acted upon remains trivial.

Note: For the Ising model TQC qubits, interferometric measurements <u>are</u> projective.

Ising vs Fibonacci

(in FQH)

Ising

- Braiding not universal
  (needs one gate supplement)
- Almost certainly in FQH
- $\Delta_{\nu=5/2} \sim 600$ mK
- Braids = Natural gates
  (braiding = Clifford group)
- No leakage from braiding
- Projective MOTQC
  (2 anyon measurements)
- Measurement difficulty distinguishing I and ψ
  (precise phase calibration)

Fibonacci

- Braiding is universal
- Maybe not in FQH
- $\Delta_{\nu=12/5} \sim 70$ mK
- Braids = Unnatural gates
  (see Bonesteel, et. al.)
- Inherent leakage errors
  (from entangling gates)
- Interferometrical MOTQC
  (2,4,8 anyon measurements)
- Robust measurement distinguishing I and ε
  (amplitude of interference)

Conclusion

- Anyons could provide a quantum computer.
- Teleportation has anyonic counterpart.
- Bounded, adaptive, non-demolitional measurements can generate the braiding transformations used in TQC.
- Stationary anyons hopefully makes life easier for experimental realization.
- FQH interferometer technology is rapidly progressing.