Seismic Noise Correlations

- RL Weaver, U Illinois, Physics
Over the last several years, Seismology has focused growing attention on Ambient Seismic Noise and its Correlations.

Citation count on one of the seminal papers:

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<th>Year</th>
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<td>2005</td>
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High-resolution surface-wave tomography from ambient seismic noise
NM Shapiro et al  *SCIENCE* **307** 1615–1618  MAR 2005
The reason is (in part) due to the striking maps of seismic velocity that noise reveals . . .

A map of Surface-Wave Velocity in California

Obtained from correlating seismic noise

Frequencies $\sim 0.02 < f < 1 \text{ Hz}; \ 3 \text{ km} < \lambda < 150 \text{ km}$
Lin and Ritzwoller and Snieder (2009) Geophys J Int
3 years of data on a bigger array
Tomographically generated maps of wave speed

Different properties at different frequencies
i.e., different depths
They even resolve ~ 1% anisotropies in wave speed
Main Assertion of Theory:

\[ G(\vec{x}, \vec{y}; \tau) \sim \frac{\partial}{\partial \tau} < \psi(\vec{x}, t)\psi(\vec{y}, t + \tau) > \]

Correlation of a diffuse field \( \psi(\vec{x}, t) \)
gives the Green's function
( + sundry fine print and qualifications)

Where did this assertion come from?
Why should we believe it?
How much should we believe it?
History of the approach . . .
Conversations with a seismologist, at a 1999 workshop,
about the seismic coda - which appeared to be equipartitioned

An earthquake record

Ray arrivals are followed by low amplitude noise, or "coda"

The coda appears to achieve a steady state ratio of its energy contents
For example, its shear-to-dilational energies: S/P

*Phys Rev Lett* 86 3447-50 (2001)
History of the approach continued. . .

I then pointed out

"if a wave field (e.g. seismic coda) is multiply scattered to the point of being equipartitioned, the field's correlations should be Green's function,

And we could recover lots of information without using a controlled source"

Geophysicist: "useful, if true"

Physicist: "Nonsense, can't possibly be true"
Hand-waving plausibility argument . . .

that there could well be a signature, an "arrival,"
at the correct travel time
- due to those few rays that happen to be going the right way
  But G exactly?
  And where's the proof?
  And won't other ray directions obscure the effect?
Standard Proofs . .

- For a thermally diffuse field
  modal picture
  fluctuation-dissipation theorem
- For a conventional acoustic diffuse field
  modal picture (sensible only for closed systems)
  plane wave picture (sensible only for homogeneous systems)
- Systems with uniformly distributed incoherent sources everywhere
- Heterogenous loss-free region without sources, but *insonified* by an external diffuse field

But what about imperfectly diffuse fields?
- There is an asymptotic ~validity to assertion
The simplest proof involves a common definition of a fully diffuse field, from room acoustics or physics of thermal phonons: in terms of the normal mode expansion for the field in a finite body

\[ \phi(x,t) = \Re \sum_{n=1}^{\infty} a_n u_n(x) \exp\{i \omega_n t\} \]

\[ <a_n a_n^*> = \delta_{nm} F(\omega_n) / \omega_n^2 \]

"equipartition"

n.b: this follows from maximum entropy
where \( F \sim \text{energy per mode (} k_B T \) \)

\[ C \equiv <\phi(x, t) \phi(y, t+\tau)> = \frac{1}{2} \Re \sum_{n=1}^{\infty} F(\omega_n) u_n(x) u_n(y) \exp\{-i \omega_n \tau\} / \omega_n^2 \]

Compare with \( G \ldots \)

\[ G_{xy}(\tau) = \sum_{n=1}^{\infty} u_n(x) u_n(y) \frac{\sin \omega_n \tau}{\omega_n} \]

[ for \( \tau > 0 \), 0 otherwise ]

So, \( \partial C / \partial \tau = G - G^{\text{time reversed}} \), i.e., \( G - G^* \) or \( \text{Im } G \) if \( F \) is constant
Verification?

\[ \lambda \ll L \approx 10 \text{ cm} \]

Hi Q aluminum block

\[ \alpha L \ll 1 \]

*J Acoust Soc Am.*, **110** (2001)

![Diagram](image)
Comparison of a Direct Pulse-Echo Signal, (conventional ultrasonics) and Thermal Noise Correlation
Correlation Of Thermal noise

$rms \ u \sim 3 \ fm/\sqrt{\text{MHz}}$

Direct Pulse-Echo Signal

After Capturing 320 seconds Of data (and taking 2.5 hours to do so)

*Phys Rev Lett* 87 134301 (2001)
But proofs that require full diffusivity and/or finite bodies and closed acoustic systems, May not be relevant for practice.

Ambient seismic noise(*), for example, is NOT fully diffuse
It has preferred directions (sources in ocean storms)

Nevertheless, these maps are impressive

Why does it work?

*Late coda appears fully diffuse, but there isn't enough of it.
Consider a homogeneous medium with incoherent sources at infinity.

What if an incident field does not have isotropic intensity? What if it is not equipartitioned?

Intensity distribution $B(\theta)$
The field in the vicinity of the origin is a superposition of plane waves

$$\psi(\vec{r}, t) = \int A(\theta) \exp(-ik\hat{\theta} \cdot \vec{r} + i\omega t) \, d\theta$$  \hspace{1cm} (2-d)

with \( < A >= 0; \quad < A(\theta)A^*(\theta') >= B(\theta)\delta(\theta - \theta') \)

i.e, an incident plane wave intensity \( B(\theta) \)

\[ \vec{r} \cdot \vec{r}' \]

Which implies that the field-field correlation is

$$< \psi(\vec{r}, t)\psi(\vec{r}', t') >= \int B(\theta) \exp(-i\omega\hat{\theta} \cdot (\vec{r} - \vec{r}') / c + i\omega(t - t')) \, d\theta$$

exact
\[ C = \langle \psi(\tilde{r}, t) \psi(\tilde{r}', t') \rangle = \int B(\theta) \exp(-i\omega \hat{\theta} \cdot (\tilde{r} - \tilde{r}') / c + i\omega (t - t')) \, d\theta \]

wavelet \( S(t) \) related to power spectrum of noise

\[
\frac{-1}{4\pi} \sqrt{\frac{2\pi}{\omega x}} \int_0^{+\infty} d\omega i \exp(i\omega(t - x / c)) \tilde{S}(\omega - \omega_o) \times \{ B(0)e^{i\pi/4} + B''(0)\frac{1}{2\omega x} e^{3i\pi/4} - B(0)\frac{i}{8\omega x} e^{5i\pi/4} \text{..} \} + \text{c.c.}
\]

Leading term \[ \Rightarrow \text{first correction} \]

Permits us to show that the apparent arrival time is delayed relative to \( |r - r'|/c \) by a fractional amount \( B''(0)/2k^2|\tilde{r}-\tilde{r}'|^2B(0) \)

\[ \Rightarrow \text{The effect of non-isotropic B or arrival time is small in practice} \]

\[ \Rightarrow \text{Hence the high quality of the maps of seismic velocity even though the ambient seismic noise is not equipartitioned} \]
Comparison of Correlation waveform (solid line) and time-symmetrized $G$ (dashed line)

For case of non-trivial ponderosity $B(\theta) = 1 - 0.8 \cos \theta$

Note:  
a) assertion fails, $G \neq dC/d\tau$

b) large differences in positive and negative time amplitudes

c) there are *tiny* shifts of apparent arrival time, as predicted

Froment et al 2009
In sum, the method works well for arrival times, hence the good maps.

The method is well suited to seismology because...

Controlled sources are highly inconvenient, (earthquakes and nuclear explosions)

Recent advent of large arrays of long-period seismic stations
and world-side access to their time records

For many years seismologists would record seismic time-records,
ignore the noise, and examine the earthquakes

Now they throw out the earthquakes and keep the noise.
Other consequences of imperfectly partitioned ambient noise:

- Spurious features in the correlations due to scatterers
- Amplitude information is hard to interpret
Correlations in the presence of a scatterer will show

a direct arrival at $\tau = |\mathbf{r} - \mathbf{r}'|/c$

an indirect arrival at $\tau = |\mathbf{r} - \mathbf{s}|/c + |\mathbf{r}' - \mathbf{s}|/c$

and

a spurious arrival at $\tau = |\mathbf{r} - \mathbf{s}|/c - |\mathbf{r}' - \mathbf{s}|/c$  \hspace{1cm} \{ parts of $G$ \}

Disappears if field is equipartitioned

Spurious arrivals..

Intensity distribution $B(\theta)$
Amplitude information?

The technique has been used very successfully in seismology to recover seismic velocities, with high spatial resolution. But . . .

Arrival time is evident

Arrival amplitude? Is this meaningful?

If we really had $G$, we'd be able to infer attenuation also. Issues include the unknown field intensity in the direction between the detectors.
Ray amplitudes $X$ depend on attenuation $\alpha$
"on-strike" intensity $B$

$$X_{i \rightarrow j} = B_i(\hat{n}_{i \rightarrow j}) \sqrt{\frac{2\pi}{\omega_o} \mid \vec{r}_i - \vec{r}_j \mid} \exp\left(-\int_{\vec{r}_i}^{\vec{r}_j} \alpha(\vec{r}) \, d\ell\right)$$

If field is not equipartitioned, then $B$ varies. But how?

Noise intensity $B$ varies in space like an RTE?

$$\hat{n} \cdot \vec{\nabla}B(\vec{r}, \hat{n}) + 2\alpha(\vec{r})B(\vec{r}, \hat{n}) = P(\vec{r}, \hat{n}) + \oint B(\vec{r}, \hat{n}')p(\vec{r}, \hat{n}, \hat{n}')d\hat{n}'$$

\text{sources} \quad \text{Scattering into direction n}
Another application

Detecting changes in a medium . . .


correlated ocean-generated seismic noise on a daily basis from an array of seismometers in Parkfield Ca.

Typical Daily Correlation between two of the stations:

\[ C_{ij}(\tau) \]

- Very hard to interpret.
- The correlations are about 80% converged.
- No clear "arrivals."
- This is $G??$
They then constructed dilation correlation coefficients $X(\varepsilon)$ (a measure of relative stretch)
(This is a 4th order statistic on the seismic field)

Between the $C_{ij}$ on different dates and the (year-long) average $C_200$

<table>
<thead>
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<tr>
<td>200</td>
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Method has been used to predict volcano eruptions
In Sum . .

It has been about 12 years now, and the topic is still growing, still hot, especially in seismology

Applications in

- High resolution seismic velocity maps
- Maps of attenuation too?
- Monitor changes in a medium
- Maps of scattering?

We still need better understanding of the effects of imperfectly diffuse fields.