Perspectives in Doped Topological Materials and Superconductivity

Pedro L S Lopes
UIUC - 10/21/2015
Take away message:
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  - vortex bound modes and vortex dynamics;
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• Topological superconductivity (doped WSM)
  - vortices, anomalies and ground states;
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- A couple of examples on vortex physics;
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  - vortex bound modes and vortex dynamics;
- Topological superconductivity (doped WSM)
  - vortices, anomalies and ground states;
- Future advances: tilted Weyl cones?
Non-interacting topological matter
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- Non-interacting gapped systems: featureless?

\[ E \]

\[ k \]
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- Topology and surface gapless modes

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Ryu et al. (2009)
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Doped Topological Insulators: Superconductivity
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- Exploit physics of defects in the novel state
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Doped TI + SC
Doped Superconducting Topological Insulators: Realization

- An example: $\text{Cu}_x\text{Bi}_2\text{Se}_3$  (SC at $x=0.12$)

Hor et al. (2009) Wray et al. (2011)
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Surface electronic behaviour strongly sensitive to doping:
• Hexagonal warping of Fermi surface of surface states
• Strong renormalisation of Fermi velocities

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Doped Superconducting Topological Insulators: vortex bound modes

- Minimal model

\[ \mathcal{H}_{kdG}^{BdG} = \begin{bmatrix}
H_{k}^{TI} - \mu & \Delta \\
\Delta^* & \mu - H_{k}^{TI}
\end{bmatrix} \]

Hosur et al. (2011) **PLSL** and Ghaemi (2015)
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\[ \mathcal{H}_l^{\text{vrtx}} = \begin{bmatrix} E_l^+ - \tilde{v}_l \partial_z^2 & -i \tilde{\Delta}_l \partial_z \\ -i \tilde{\Delta}_l \partial_z & -E_{-l}^+ - \tilde{v}_l \partial_z^2 \end{bmatrix} \]

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Doped Superconducting Topological Insulators: Measuring the vortex phase transition

- DOS schematics
Doped Superconducting Topological Insulators: Measuring the vortex phase transition

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![Graph showing the SC gap](image)
Doped Superconducting Topological Insulators: Measuring the vortex phase transition

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![Graph showing DOS schematics with SC gap and CdG mini-gap](image)
Doped Superconducting Topological Insulators: Measuring the vortex phase transition

- Enhance signatures by looking at effects from vortex quantum dynamics

\[ \Delta (\mathbf{r} - \mathbf{R}(\tau)) \approx \Delta (\mathbf{r}) - \partial_{\mathbf{r}} \Delta (\mathbf{r}) \cdot \mathbf{R}(\tau) \]

\[ S_{\text{vrtx}} = \frac{m_v}{2} \int \frac{d\omega}{2\pi} \mathbf{R}^\dagger(i\omega) \begin{pmatrix} \omega^2 + \omega_0^2 & \omega_c \omega \\ -\omega_c \omega & \omega^2 + \omega_0^2 \end{pmatrix} \mathbf{R}(i\omega) \]

Bartosch and Sachdev (2006)
Doped Superconducting Topological Insulators: Measuring the vortex phase transition

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Harmonic trap

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- Magnus force
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- New energy scales!
- Study how these renormalize bound modes energies

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Doped Superconducting Topological Insulators: Measuring the vortex phase transition with LDOS

- Enhance signatures by looking at effects from dynamics

\[ \omega - E_{l}^{\pm} - \Sigma_{l}^{\pm}(\omega) = 0 \]
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New energy scale

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Superconducting Doped Topological Insulators

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Superconducting Doped Topological Insulators

Summary: vortices in doped TIs

- Vortex phase transition;
- Radially dependent LDOS profile for vortex;
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Learning how to project the vortex mode into a Kitaev chain allows one to envisage how to apply this mechanism in 1D scenarios where energy scales might be more friendly

PLSL and Ghaemi (2015)
Topological Superconductivity (TRI)

- TSC: Class DIII in 3+1D
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\[
H = \frac{1}{2} \sum_{\mathbf{k}} \begin{pmatrix} c_{\mathbf{k}}^\dagger, c_{-\mathbf{k}} \end{pmatrix} \begin{pmatrix} h_{\mathbf{k}} & \Delta_{\mathbf{k}}^\dagger \\ \Delta_{\mathbf{k}} & -h_{\mathbf{-k}}^T \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}} \\ c_{\mathbf{-k}}^\dagger \end{pmatrix}
\]

\[
\begin{array}{cccc|ccc}
\text{Symmetry} & \text{AZ} & \Theta & \Xi & \Pi & 1 & 2 & 3 \\
A & 0 & 0 & 0 & 0 & Z & 0 & Z \\
AIII & 0 & 0 & 1 & Z & 0 & Z & 0 \\
AII & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
CII & 0 & 1 & 0 & Z & Z & 0 & 0 \\
D & 1 & 1 & 1 & Z & 0 & 0 & 0 \\
DIII & -1 & 1 & 1 & Z & Z & Z & Z \\
AIII & -1 & 0 & 0 & 0 & Z & Z & Z \\
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h_k = \frac{k^2}{2m} - \mu + \alpha \sigma \cdot k
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\[
\Delta_k = i\Delta_0 \sigma_y \sigma \cdot k.
\]
Doped Weyl SM:
Platform for Topological SC

• 2 Fermi surfaces: project!
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- Minimal model = SC gapped doped Weyl semi-metals

\[ H = \sum_i \psi_i^\dagger (v_i \mathbf{p} \cdot \mathbf{\sigma} - \mu_i) \psi_i + \Delta_i \psi_i \sigma_y \psi_i + H.c. \]
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\]

• Topological invariant

\[
N = \frac{1}{2} \sum_i C_{1i} \text{sgn} (\Delta_i)
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Qi et al. (2010)
Class DIII TSC: Effective action, chiral vortices, anomalies

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SC phases from each Fermi surface must deconfine at surfaces

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S_{\text{eff}} [A] = \int_x \left[ \frac{1}{2} \frac{\varepsilon^\sigma_{\mu \lambda \nu}}{8\pi^2} \partial_\sigma (\theta_L - \theta_R) A_\mu \partial_\lambda A_\nu + J \cos (\theta_L - \theta_R) \right]
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- Surfaces of TSCs may bind vortices in the phase of a single Fermi surface

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\[ \text{Chiral vortices} \]

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Class DIII TSC: Incurable anomalies?!

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- Majorana chiral modes 

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\[ \langle J^\mu \rangle = \frac{1}{2} \frac{e}{8\pi^2} \epsilon^{\mu\nu\lambda\rho} \partial_\nu (\theta_R - \theta_L) F_{\lambda\rho} \]
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- But vortex modes are neutral! (no Callan-Harvey)
Class DIII TSC: Cooper pair anomalous pumping?

• A possible escape route: condensate reabsorption of pumped charge

\[ \partial_{\mu} \langle J^\mu \rangle = \frac{1}{2} \frac{e}{2\pi} \left[ \delta^2 (\mathbf{x}) \right] F_{0z} \]
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- Majorana chiral modes are not pumped by electric field but gather non-trivial phases upon vortex linking
Class DIII TSC: Vortex linking and JJs

• Linking of a pair of chiral Majorana loops is equivalent to evolving the SC phase across a QSH edge Josephson junction

Single squashed Majorana loop description:

Fendley et al. (2009)
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\[ H_{loop} = \int_0^{2\pi} d\theta \gamma i \partial_x \gamma \]
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\[ L_b = -iav\gamma_{1L}(0) \gamma_{1R}(0) + ibv\gamma_{1L}(L) \gamma_{1R}(L) \]

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Double squashed Majorana loop description:

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\]

\[
H_{2\text{loop}} = v \frac{1}{2} \int d^2 x \Psi^\dagger \left[ -i \rho_z \tau_z \partial_x + \rho_x \Delta \right] \Psi
\]

\[
\Delta = e^{-i\varphi_L} \delta(x - L) - e^{-i\varphi_0} \delta(x)
\]

\[
\Psi = \left( \psi_R, \psi_L, -\psi_L^\dagger, \psi_R^\dagger \right)^T
\]
Class DIII TSC: Vortex linking and JJs

- JJs+QSH edge+interactions = $Z_4$ fractional J effect

- Results can be almost directly imported to our case

Zhang, Kane (2014)
Class DIII TSC: Summary

- Interplay of SC and multiple topological Fermi surfaces;
- Real space surfaces may deconfine the SC phases from the different FSs;
- Anomalous EM;
- Only cure: Cooper pair pumping;
- Anomalies, vortex linking and JJs at QSH edges are intertwined in this problem;
Future prospects:
New topological platforms for SC
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• A little speculation: tilted Weyl cones
Future prospects: New topological platforms for SC

• A little speculation: tilted Weyl cones

Soluyanov et al. (2015)
Future prospects: New topological platforms for SC

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\[ H(k) = \sum_{i,j} k_i A_{ij} \sigma_j \]
\[ i = 1, 2, 3 \]
\[ j = 0, 1, 2, 3 \]
Future prospects: New topological platforms for SC

- A little speculation: tilted Weyl cones

\[ H(\mathbf{k}) = \sum_{i,j} k_i A_{ij} \sigma_j \]

\( i = 1, 2, 3 \)
\( j = 0, 1, 2, 3 \)

- Weyl points are located at the touching points of electron-hole pockets
- Finite density of states even if chemical potential tuned to the Fermi point. (stronger screening than in regular WSM)

Soluyanov et al. (2015)
Conclusions
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- SCs with normal phases stemming from doped TIs or WSMs display unusual physics for electronic modes bound to vortices;
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Conclusions

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• For TSCs, axionic effective field theories imply gauge violations which cannot be canceled by standard arguments. Possible ground state degeneracies may solve this enforcing quantum pumping of Cooper pairs;
• A new generation of metallic systems is being developed in which novel SC phases might find new platforms to realize;
Class DIII TSC: Vortex linking and JJs

\[ \varphi = \pi \]

\[ \varphi = \frac{\pi}{2} \]

\[ \varphi = 0 \]
Class DIII TSC: Vortex linking and JJs

- Different superconducting phases across the JJ indeed correspond to different boundary conditions;
Class DIII TSC: Vortex linking and JJs

• Different superconducting phases across the JJ indeed correspond to different boundary conditions;
• Many body spectrum of the JJ imply 4-fold periodicity as function of the SC phase difference at the JJ in the presence of interactions.
Class DIII TSC: Vortex structures

\[ \Delta \theta = \pi \quad \Delta \theta = 0 \]
Non-interacting topological matter: Classification

• Topological defects and other gapless modes

\[ h_{ij} \rightarrow \mathcal{H}(k, r) \]
Non-interacting topological matter: Classification

- Topological defects and other gapless modes

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$$ h_{ij} \rightarrow \mathcal{H}(k, r) $$
Non-interacting topological matter: Classification

• Gapless bulk: protected Fermi surfaces

\[ p = d - d_{FS} \]
Non-interacting topological matter: Classification

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Chiu and Schnyder (2014)