TOPOLOGY IN CONDENSED MATTER SYSTEMS: MAJORANA MODES AND WEYL SEMIMETALS

Pavan Hosur  
UC Berkeley

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Acknowledgements

Advisor:
Ashvin Vishwanath
UC Berkeley

Collaborators:
Pouyan Ghaemi
UC Berkeley → UIUC

Roger Mong
UC Berkeley

Sid Parameswaran
UC Berkeley
Introduction: General examples of topology in condensed matter

Focus example 1: Majorana modes using topological insulators and superconductors

Focus example 2: Weyl semimetals – introduction and transport
Outline

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Topology in real space

Topological defects

- Domain walls — charge density wave, quantum magnets
- Vortices — type-II superconductors, superfluids
- Hedgehogs — quantum magnets, valence bond solids
Topological band structures

- Integer quantum hall state (Chern insulator)
- Topological insulators
- Topological superconductors (e.g. He-3 B)

Singularities in dispersion

- Fermi surface
- Line nodes (e.g. graphite)
- Point nodes (e.g. Graphene, Weyl semimetals)
Combining real and momentum space topologies

- Fermionic Hopf skyrmion in topological insulator-superconductor system [Ran, PH, Vishwanath ‘11]
- Vortex in $p+ip$ superconductor [Read-Green’00]
- Vortex in superconductor on topological insulator surface [Fu-Kane’08]

Carry Majorana zero modes
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- Focus example 2: Weyl semimetals – introduction and transport
Crash-course on topological insulators: surface states

- Bulk insulators; surface states traversing bulk gap protected by time-reversal symmetry
- Surface states “relativistic” at low energies
- $\pi$–Berry phase around the Fermi surface
Crash-course on topological insulators: 
\( \mathbb{Z}_2 \) invariant \( \nu_0 \)

\[ \nu_0 = \begin{cases} -1 & \Rightarrow \text{topological insulator} \\ 1 & \Rightarrow \text{trivial insulator} \end{cases} \]

Surface: \( \nu_0 = (-1)^{\text{number of Dirac nodes}} \)

Inversion symmetric bulk: \( \nu_0 = \prod_{k \in \text{TRIM}} P_{\text{occ}}(k) \)

\( \text{TRIM} = \text{time-reversal invariant momenta} \)

\( P = \text{parity eigenvalue} \)

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\( \nu_0 \)

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\( \text{Trivial insulator} \)

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\( \text{Topological insulator} \)

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\( \text{p-band} \)

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\( \text{s-band} \)

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\( \text{band inversion} \)
Crash-course on topological insulators: real materials

- Bi-Sb alloy [Fu’07, Hsieh’09]
- Bi$_2$X$_3$, X=Se,Te [Chen’09, Hsieh’09…]
- Tl-chalcogenides [Yan’10,…]
- Half-heusler compounds [Xiao’10,…] and many more
Majorana fermions in superconductors

- Majorana fermion: particle that is its own anti-particle
- Bogoliubov-de Gennes Hamiltonian has inherent $E \rightarrow -E$ symmetry

Majorana zero mode
(Protected by "particle-hole" symmetry)
Systems with Majorana zero modes

Vortices in...

- $p+ip$ superconductor [Read, Green '00]
- Superconductor-topological insulator interface [Fu, Kane '08]
- Semiconductor-superconductor heterostructures [Sau et. al. '10, Lutchyn et. al. '10, ...]
- Possibly, 5/2 fractional quantum hall state [Read-Green'00, Moore-Read’91]
Majorana mode at superconductor-topological insulator interface

$\phi = \hbar/2e$

Zero energy Majorana mode

s-wave superconductor

Strong Topological Insulator

Fu-Kane PRL 2008
Majorana mode in superconducting-doped topological insulator?

\[ \phi = h/2e \]

Superconductivity in Bi\textsubscript{2}Te\textsubscript{3}

Zhang et al. PNAS ‘11

Yes! But only if…

PH, Ghaemi, Mong, Vishwanath, PRL’11
Fermi level in normal phase, which has a band inversion.

Evolution of vortex, surface Majorana modes.

Majorana mode present.

Majorana modes from opposite surfaces annihilate.

Upto what $\mu$ do Majorana modes survive?

PH, Ghaemi, Mong, Vishwanath, PRL’11
At what $\mu$ does the vortex have two bulk Majorana zero modes?
Towards main result: Analogy with spinless $p+ip$ superconductor at fixed $k_z$

Superconducting doped topological insulator

$$H_{BdG} = \begin{bmatrix} H_k - \mu & \Delta(r) \\ \Delta^*(r) & \mu - H_k \end{bmatrix}$$

Vortex: \( \Delta(r) = \Delta_0(x + iy) \)
\( \equiv \Delta_0(i\partial_{k_x} - \partial_{k_y}) \)

Has Majorana zero mode at $\pi$-flux if $\varepsilon(r_0) = 0$ for some $r_0$

PH, Ghaemi, Mong, Vishwanath, PRL'11

Spinless $p+ip$ superconductor

$$H_{p_x+ip_y} = \begin{bmatrix} \varepsilon(r) & \Delta(i\partial_x - \partial_y) \\ \Delta(i\partial_x + \partial_y) & -\varepsilon(r) \end{bmatrix}$$

Fermi surface has $\Box H_k - \mu \Box = 0$. Expect a Majorana zero mode at $\pi$-Berry phase

Read-Green, PRB 2000
Main result: $SU(2)$ Berry phase and $\mu_c$

- Bands doubly degenerate (Kramer’s degeneracy)
- Berry connection non-Abelian, 2x2 matrix

$$A_{ij}(k) = i \langle u_i(k) | \nabla_k u_j(k) \rangle; \ i, j \in \{1,2\}$$

- At each $k_z$, Berry phase factor around Fermi surface

$$U_B(k_z) = P \left[ \exp \left( i \oint \hat{A}(k) \cdot \hat{z} \right) \right] \in SU(2)$$

$P = \text{path-ordering}$;
$U(1)$-part vanishes due to time-reversal+inversion;

PH, Ghaemi, Mong, Vishwanath, PRL’11
Main result: $SU(2)$ Berry phase and $\mu_c$

eigenvalues $(U_B(k_z)) = \{e^{i\phi(k_z)}, e^{-i\phi(k_z)}\}$

$$E_{BdG}(k_z) = E_0[(2n + 1)\pi - \phi(k_z)]$$

has zero mode if $\phi(k_z) = \pi$

$$\phi(k_z) = \phi(-k_z)$$

$\Rightarrow$ 0, 4, 8,... zero modes at $k_z \neq 0$

$\Rightarrow$ sufficient to focus on $k_z = 0, \pi$

$E_0 \sim \Delta^2 / \mu$

PH, Ghaemi, Mong, Vishwanath, PRL’11
Among doped topological insulators:

- $p$-doped $\text{TlBiTe}_2$ [Hein '70]
- $p$-doped $\text{Bi}_2\text{Te}_3$ under pressure [J. L. Zhang '11]
- $n$-doped $\text{Bi}_2\text{Te}_3$ [Hor '10]
- $\text{Cu-Bi}_2\text{Se}_3$ [Hor '10, Wray'10] near phase transition for $c$-axis vortex, but has Majorana modes for $ab$-axis vortex

Among ordinary insulators:

- Either $\text{PbTe}$ or $\text{SnTe}$ and $\text{GeTe}$ ($\text{PbTe}$ has four band inversions relative to $\text{SnTe}$ and $\text{GeTe}$)

PH, Ghaemi, Mong, Vishwanath, PRL’11
Surface Majorana zero modes present in vortex of superconducting doped topological insulator below critical doping determined by Berry phase of normal state Fermi surface

Several existing superconductors expected to carry vortex Majorana modes, possibly even non-topological insulator-based ones
Future questions

- What if parent insulator breaks inversion?
- What about other geometries such as domain walls?
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Weyl semimetals

3D materials with linear band-touchings

Weyl nodes “topological objects” in momentum space

Dirac equation

\[ H = \hat{\alpha}_x p_x + \hat{\alpha}_y p_y + \hat{\alpha}_z p_z + \beta m \]

\[ \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \]

Weyl equation

\[ H = \hat{\sigma}_x p_x + \hat{\sigma}_y p_y + \hat{\sigma}_z p_z \]

\[ \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \]

No mass term possible in Weyl Hamiltonian! (unlike graphene)
Why study Weyl semimetals?

- Fermi arc surface states
  - Wan et. al. PRB 2011

- Chiral Anomaly
  - Nielsen-Ninomiya PLB 1983

- Surface states of 4D Chern insulator
  - Zhang-Hu Science 2001

- Real material candidates:
  - Pyrochlore iridates
  - TI-FM multilayer
  - HgCr$_2$Se$_4$
  - Wan PRB’11, Burkov PRL’11, Fang PRL’11
Transport in Weyl semimetals

\[ \sigma = \frac{e^2}{h} [L]^{-1} \]

With Coulomb interactions
- \( \omega \ll \alpha^2 T \) transport collision-dominated
- Solve quantum Boltzman equation within ‘leading-log approximation’

With chemical potential disorder
- Assume point scatterers
- Use Born approx. for self-energy
- Conductivity using Kubo formula

PH, Parameswaran, Vishwanath, arXiv:1109.6330
Coulomb transport: mechanism

**Generic Fermi surface**

\[ j \neq 0, \; k_{tot} \neq 0 \]

No relaxation to \( k_{tot} = 0 \) state without disorder

\[ \Rightarrow \text{No current!} \]

**Dirac dispersion at neutrality point**

\[ j \neq 0, \; k_{tot} = 0 \]

Interactions, even though they conserve momentum, sufficient to relax current
Coulomb transport: calculation

Quantum Boltzmann analysis

- \( j(t) = ev_F \sum_{\lambda=\pm} \int k \hat{\lambda} f(k,t) = \sigma E(t) \)

- **QBE:** \( (\partial_t + eE \cdot \nabla_k) f(k,t) = w[f(k,t)] \) \( w = \) scattering rate; use Fermi’s golden rule

Fritz et. al., PRB 2008
Quantum Boltzmann analysis contd.

- Rewrite integral equation as variational problem.

- \( V_{\text{Coulomb}}(q) \sim \frac{1}{q^2} \Rightarrow \text{small-}\( q \text{ processes dominate}

- Leading-log approximation:

\[
\int_0^\infty \frac{dq}{q} \rightarrow \int_{T\alpha}^{T} \frac{dq}{q} = \ln \alpha^{-1}, \quad q = 0 \text{ elsewhere}
\]

- Variational ansatz eigenfunction of leading-log term.
Coulomb transport: results

\[ \sigma = N \frac{e^2}{h} \frac{1.8}{-i6.6 \frac{\hbar \omega}{k_B T} + N \alpha_T^2 \ln \alpha_T^{-1}} \left( \frac{k_B T}{\hbar v_F(T)} \right) \]

DC limit interpretation

- \[ \sigma = e^2 D \frac{dn}{d\mu} \quad \text{...thermally excited electron-hole plasma} \]
- \[ D = v_F^2 \tau, \quad \hbar \tau^{-1} \sim N \alpha^2 T, \quad \frac{dn}{d\mu} = \frac{T^2}{v_F^3} \]
- \[ \Rightarrow \sigma_{dc} \sim \frac{e^2}{h} \frac{1}{\alpha_T^2} \frac{T}{v_F(T)} \]

Use renormalized \( v_F \) and \( \alpha \); e un-renormalized due to gauge invariance

PH, Parameswaran, Vishwanath, arXiv:1109.6330
Coulomb transport and experiments

\[ \sigma_{dc} = \frac{e^2}{h} \frac{1.8}{\alpha_T^2 \ln \alpha_T^{-1}} \left( \frac{k_B T}{\hbar v_F(T)} \right) \]

1. Polycrystalline Y$_2$Ir$_2$O$_7$

Yanagishima-Maeno, JPSJ 2001

Fit to theory – Coulomb interactions-driven resistivity

PH, Parameswaran, Vishwanath, arXiv:1109.6330

Y$_2$Ir$_2$O$_7$
Coulomb transport and experiments

\[ \sigma_{dc} = \frac{e^2}{h} \frac{1.8}{\alpha_T^2 \ln \alpha_T^{-1}} \left( \frac{k_B T}{\hbar \nu_F(T)} \right) \]

2. Single crystal Eu$_2$Ir$_2$O$_7$

Low pressure curves show agreement with theory

Tafti et.al. arXiv:1107.2544
Transport with disorder

- Born-approximation for self-energy
- Kubo formula for conductivity

\[ \sigma_{dc} = N \frac{e^2}{\gamma h} \]

Order of limits \( \omega \to 0 \) and \( T \to 0 \) crucial!

PH, Parameswaran, Vishwanath, arXiv:1109.6330
Weyl semimetals transport summary

With interactions

- Quantum Boltzmann equation solved within leading-log approx.
- $\sigma_{dc} \sim T$ resembles experimental data

With disorder (strength $\gamma$)

- Different behavior for $\omega << T$ and $\omega >> T$
  - $\omega << T$: $\gamma^{-1}$ Drude peak of width $\sim T^2$
  - $\omega >> T$: $\sigma \sim |\omega|$
Future research

- Quantum oscillations due to Fermi arcs (in progress)
- Signatures of chiral anomaly
- Weyl semimetals and superconductivity
- Materials challenge – better cleavable materials
  
  ...and more
Other projects

**Bosons in Kagome optical lattice**
Jo, Guzman Thomas, PH, Vishwanath, Stamper-Kurn
arXiv: 1109.1591

**Fermionic Hopf skyrmion in topological insulator-superconductor**
Ran, PH, Vishwanath, PRB 84, 184501 (2011)

**Photocurrent on topological insulator surface**
PH, PRB 83, 035309 (2011)

**3D Topological phases and topological defects form the Dirac limit**
PH, Ryu, Vishwanath, PRB 81, 045120 (2010)
Thank you

References

1. Majorana modes at the ends of superconductor vortices in doped topological insulators
   PH, Ghaemi, Mong, Vishwanath, PRL 107, 097001 (2011)
   Viewpoint by Taylor Hughes: Majorana fermions inch closer to reality, Physics 4, 67 (2011)

2. Transport in Weyl semimetals
   PH, Parameswaran, Vishwanath, arXiv: 1109.6330 (To appear in PRL)
Bulk-surface band intersection and $\mu_c$

- **Minimal model:**
  $$H = v_D \tau_z \sigma \cdot k + \tau_x m(k)$$

- Surface and bulk bands intersect at
  $$E = v_D k$$
  and Majorana modes can “leak in”

- **At** $E_{bulk} = v_D k$, $m(k) = 0$

- **At** $m(k) = 0$, **Berry phase** $= \pi$ (per $k_z = 0$ Fermi surface)
Crash-course on topological insulators: \( \nu_0 \) with inversion symmetry

\[ \nu_0 = P(000)P(00\pi) = (1)(1) = 1 \]

...trivial insulator

\[ \nu_0 = P(000)P(00\pi) = (-1)(1) = -1 \]

...topological insulator

(Assuming no band inversions at other TRIMs)
Another perspective: Topological phase transition inside a topological defect

- Vortex = 1D system in class D (BdG with no time-reversal or spin-rotation symmetry)
- $\mathbb{Z}_2$ invariant signals presence/absence of end Majorana modes [Kitaev’01]

Evolution of vortex, surface Majorana modes

Evolution of vortex dispersion

PH, Ghaemi, Mong, Vishwanath, PRL’11