Quantum Hall Effect in Graphene p–n Junctions

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January 14, 2008
Electron transport in graphene monolayer

New 2d electron system (Manchester 2004):
Nanoscale system with tunable transport properties;

Field-effect enabled by gating:
cconductivity linear in density, mobility, density vs gate voltage

Andrey Geim
Monolayer graphene

Philip Kim

Novoselov et al, 2004, Zhang et al, 2005
Electronic Properties

Semimetal (zero bandgap)

Massless Dirac electrons

1) Two species of Dirac fermions – valleys K, K';

2) Components of Dirac spinor=amplitudes on sublattices A and B.
The “half-integer” QHE in graphene

Single-layer graphene:
QHE plateaus observed at
\[ \nu = 4 \times (0, \pm 1/2, \pm 3/2 \ldots) \]
4=2x2 spin and valley degeneracy

Manifestation of Dirac nature of excitations!

Double-layer (non-Dirac):
\[ \sigma_{xy} (4e^2/h) \]

Particle-hole symmetry

Novoselov et al, 2005, Zhang et al, 2005
QHE in graphene: Landau levels

\[ E_n = \text{sgn}(n)|n|^{1/2}\varepsilon_0, \quad \varepsilon_0 = \hbar\nu_0 \left(\frac{2eB}{\hbar c}\right)^{1/2} \]

Particle-hole symmetric; has a zero mode

\[ E_n \propto \sqrt{n}, \sqrt{B} \]

Separation between low-lying LL is very large, 1000 K at \( B = 10 \, \text{T} \) \( \longrightarrow \) room temperature QHE

To obtain LL's use relation \( H_{\text{Pauli-Schroedinger}} = 2m(H_{\text{Dirac}})^2 \)
Some interesting aspects of QHE in graphene

1) Electron-hole symmetric zeroth LL;

2) Atomically sharp edge;

3) Valley+spin spontaneous symmetry breaking (similarity to QH bilayers);

4) Large gaps between Landau levels --> room-T QHE

5) Possibly new electron QHE phases (fractional, stripe...
Plan

1) Quantum Hall edge states in graphene

2) Spin-filtered edge states at the Dirac point: transport properties

3) Fractional conductance quantization in gated graphene devices
QHE: edge transport (reminder)

Measurements – quantum Hall effect

**Integer QHE and Edge States**

Chiral dynamics along edge (unidirectional)

Edge conduction

No backscattering along same edge

\[ I = \sum_{\text{occ.}} i_k = \int \frac{L_x}{2\pi \ell^2} \left( \frac{\partial \epsilon_N}{\partial y_k} \frac{e \ell^2}{h L_x} \right) = \frac{e}{h} (\mu_R - \mu_L) = \frac{e^2}{h} V_H \]
Graphene QHE: edge states

- Counter-propagating electron and hole states;
- Symmetric splitting of n=0 LL
- Universality, same for other edge types;
- The odd numbers of edge modes result in half-integer QHE

Edge states from 2d Dirac model
Also: Peres, Guinea, Castro-Neto, 2005, Brey and Fertig, 2006
Spin-filtered edge states for Zeeman-split Landau levels

Near $\nu=0$, $E=0$:

(i) Two chiral counter-propagating edge states with opposite spin polarizations

(ii) No charge QHE, but quantized spin Hall effect.

Estimate of the spin gap

Exchange in spin-degenerate LL's at ν=0, E=0:
- Coulomb interaction favors spin polarization;
- Fully antisymmetric spatial many-electron wavefunction;
- Spin gap dominated by the exchange somewhat reduced by correlation energy:

\[ \Delta = \frac{n}{2} \int \frac{e^2}{\epsilon r} \left( 1 - e^{-r^2/2l_B^2} \right) d^2r = \left( \frac{\pi}{2} \right)^{1/2} \frac{e^2}{\epsilon l_B} (1 - \alpha) \]

Gives spin gap \(~100K\) much larger than Zeeman energy (10K)

Exchange-enhanced gap not observed (Yet?)
Spintronics in graphene: chiral spin edge transport

A 4-terminal device, full spin mixing in contacts

Charge current

\[ I^c_k = \sum_{k'} g_{kk'} (V_k - V_{k'}) \]

(Landauer-Buttiker)

Spin current

\[ I^s_k = \sum_{k} I^s_{kk'} = \sum_{k'} \epsilon_{kk'} g_{kk'} (V_k - V_{k'}) \]

where \( \epsilon_{kk'} = -\epsilon_{k'k} \) equals +1 (−1) when the current from \( k \) to \( k' \) is carried by spin up (spin down) electrons.

In an ideal clean system (no inter-edge spin-flip scattering):
charge current along \( V \), spin current transverse to \( V \):

\[ \rho_{xx} = \frac{h}{2e^2} \]

Quantized spin Hall conductance
Spin-filtered transport

Asymmetric backscattering filters one spin polarization, creates longitudinal spin current:

Applications: (i) spin injection; (ii) spin current detection.

Hall voltage measures spin not charge current!

Applications: (i) spin injection; (ii) spin current detection.
Spin-flip scattering of edge states

Can be due to potential edge disorder+SO:

--Spin-orbit: intrinsic+Rashba

- Only Rashba term (~1K) is effective

- Edge disorder with length scale $d \sim 1 \text{nm}$

- Gives mean free path:

$$\ell(\epsilon) \sim \left(\frac{\epsilon}{\lambda_R}\right)^2 \left(\frac{\ell_B}{d}\right)^2 d, \quad |\epsilon| \gtrsim \lambda_R,$$

where $\epsilon$ is detuning from crossing

- Typical values $l \sim 10 \mu\text{m}$ at $\epsilon \sim 10 \text{ K}$ comparable to sample size
What symmetry protects gapless edge states?

Gapless states, e.g. spin-split  
Gapped states, e.g. valley-split

Special $\mathbb{Z}_2$ symmetry requirements (Fu, Kane, Mele, 2006): in our case, the $\mathbb{Z}_2$ invariant is $S_z$ that commutes with $H$;
Manifestations in transport near the neutrality point

Gapless spin-polarized states:

a) Longitudinal transport of 1d character;
   b) Conductance of order $e^2/h$
   at weak backscattering (SO-induced spin flips);
   c) No Hall effect at $\nu=0$

Gapped states:

a) Transport dominated by bulk resistivity;
   b) Gap-activated temperature dependent resistivity;
   c) Hopping transport, insulator-like T-dependence
   d) Zero Hall plateau
Chiral spin edge states summary


- Counter-propagating states with opposite spin polarization at $\nu=0$, $E=0$;
- Large spin gap dominated by Coulomb correlations and exchange
- Gapless edge states at $\nu=0$ present a constraint for theoretical models
- Novel spin transport regimes at the edge (no experimental evidence yet)
QHE in p-n and p-n-p lateral junctions:

Edge state mixing --> Fractionally-quantized QHE

Gated graphene devices

Local density control (gating) --> p-n and p-n-p junctions

(Stanford, Harvard, Columbia)

--Deposit thin layer of dielectric on the surface
--Local gate on top, ~30nm above the sample

Equal or opposite polarities of charge carriers in the same system (electrons and holes coexist)
Fractional QHE in p-n junctions

Integer and fractional conductance quantization:
(i) $g=2, 6, 10...$, unipolar regime, (ii) $g=1, 3/2...$, bipolar regime

Williams, DiCarlo, Marcus, Science 28 June 2007
**QHE in p-n junctions: bipolar regime**

- Edge states from reservoirs propagate together along p-n boundary.

- Assume full mixing ---> resistors in series

\[
g_{pn} = \frac{|\nu_1| |\nu_2|}{|\nu_1| + |\nu_2|} = 1, \frac{3}{2}, 3, \frac{5}{3} ... \]

Explains observed fractional values
QHE in p-n junctions: unipolar regime

- Some edge states propagate through the entire system
- Conductance determined by the smaller density:

\[ g_{nn} = g_{pp} = \min(|\nu_1|, |\nu_2|) = 2, 6, 10... \]

Predicted conductance pattern agrees with experiment

- Combine with result for bipolar regime,

\[ g_{pn} = \frac{|\nu_1| |\nu_2|}{|\nu_1| + |\nu_2|} = 1, \frac{3}{2}, 3, \frac{5}{3}... \]

QHE in p-n junctions: UCF suppressed

- Similar to chaotic quantum dots, but no UCF

- Several possibilities: decoherence, thermalization, energy relaxation, self-averaging (e.g., noisy gates)

Which transport mechanism is realized?

Shot noise:
- quantized and nonzero for p-n
- no noise for p-p or n-n

Quantized shot noise (fractional F=S/I)

Fractional QHE in p-n-p junctions

\[ G = \left( \frac{e^2}{h} \right) |\nu'|. \quad |\nu'| \leq |\nu| \]

\[ G = \frac{e^2}{h} \frac{|\nu'||\nu|}{2|\nu'| - |\nu|} = \frac{6}{5}, \frac{10}{9}, \frac{30}{7}, \ldots \quad (|\nu'| \geq |\nu|) \]

\[ G = \frac{e^2}{h} \frac{|\nu'||\nu|}{2|\nu'| + |\nu|} = \frac{2}{3}, \frac{6}{5}, \frac{6}{7}, \ldots \quad (\nu \nu' < 0). \]

Little or no mesoscopic fluctuations

Stability of fractional plateaus: model

Why are some fractional plateaus more robust?

Gating central region can lead to backscattering ---> nonzero $\sigma_{xx}$

The model (based on Rendell, Girvin, PRB 23, 6610 (1985))

Dissipation only in the central region

\[ -j_x + i j_y = \exp(f(z)) \]

at interfaces: \[ \text{Im } f(z) = \arctan \left( \frac{\rho^0_{xy} - \rho^c_{xy}}{\rho^c_{xx}} \right) \]

solve using conformal mapping
Stability of fractional plateaus: results

2D transport vs edge transport: results are identical at $\sigma_{xx}=0$

Plateaus with small $\nu'$ less stable w.r.p.t. finite $\sigma_{xx}$ than other plateaus
(in agreement with experiment)

In experiment conductance at $v=v'=-2$ is $\sim 1.7e^2/h$
(no dissipation gives $2e^2/h$);

Extract diagonal conductivity $\sigma_{xx} \sim 0.5e^2/h$ in the central region using our model;

DA, Levitov, in prep.
Summary

- QHE in p-n junctions interpreted in terms of edge transport
- Fractional conductance plateaus, resulting from edge states mixing at p-n interfaces
- Manifestations of mixing: noiseless vs. noisy transport
- Stability of fractional plateaus understood in terms of backscattering in the gated region
Spin-flip scattering of edge states

Can be due to potential edge disorder + SO:

-- Spin-orbit:

\[ H_{SO} = \lambda_{SO} \sigma_z \tau_z s_z, \quad H_R = \lambda_R (\sigma_z \tau_z s_y - \sigma_y s_x), \]

- Only Rashba term (~1K) is effective

- Edge disorder with length scale \( d \sim 1 \text{nm} \)

- Gives mean free path:

\[ \ell(\varepsilon) \sim (\varepsilon / \lambda_R)^2 (\ell_B / d)^2 d, \quad |\varepsilon| \gtrsim \lambda_R, \]

where \( \varepsilon \) is detuning from crossing

- Typical values \( l \sim 10 \mu\text{m} \) at \( \varepsilon \sim 10 \text{K} \) comparable to sample size
Luttinger liquid on graphene edge: the problem (in progress)

Effect of Coulomb interactions on edge modes?

1) Left-movers carry spin up, right-movers spin down, described by general Hamiltonian:

\[ H = \int dx \left[ (\partial_x \varphi_L, \uparrow)^2 + (\partial_x \varphi_R, \downarrow)^2 + V_{12} \partial_x \varphi_L, \uparrow \partial_x \varphi_R, \downarrow \right] \]

2) Novel type of Luttinger liquid;

3) What are spin and charge conductances? Are they universal?

4) Effects of disorder? Similarity to edge reconstruction at 2/3 fractional QHE state?

......work in progress....

Related to quantum SHE edge states by Kane&Mele, but gap here is large, ~100 K
Luttinger liquid on graphene edge: approach (in progress)

How can one obtain the Luttinger Hamiltonian explicitly?

Simplify the problem: consider domain wall between spin-up and spin-down polarized states:

- Order parameter varies smoothly from $\theta=0$ to $\theta=\pi$;
- the polar angle $\phi$ is arbitrary;
- gapless charged textures when $\phi$ varies from 0 to $2\pi$;
- charge $e$! (spin sphere covered once, similar to skyrmions)
The half-integer quantization from Berry's phase

Quasiclassical Landau levels (nonrelativistic): Bohr-Sommerfeld quantization for electron energy in terms of integer flux $\Phi = n\Phi_0$ enclosed by a cyclotron orbit

For chiral massless relativistic particles (pseudo)spin is parallel to velocity, subtends solid angle $2\pi$ upon going over the orbit, thus quantization condition modified as

$$\Phi = (n+1/2)\Phi_0$$

Half-period shift of Shubnikov-deHaas oscillation

Translates into half-integer QHE in quantizing fields
“Half-integer” Quantum Hall Effect

Single-layer graphene: QHE plateaus observed at

\[ \nu = 4 \times (0, \pm 1/2, \pm 3/2 \ldots) \]

4=2x2 spin and valley degeneracy

Manifestation of relativistic Dirac electron properties

Landau level spectrum with very high cyclotron energy (1000K)

Recently: QHE at T=300K

Novoselov et al, 2005, Zhang et al, 2005
Spin filtered edge states

Dissipative Quantum Hall effect

DA et al., PRL 98, 196806 (2007)
Dissipative QHE near $\nu=0$

Longitudinal and Hall resistance, $T=4\text{K}$, $B=30\text{T}$

Features:

a) Peak in $\rho_{xx}$ with metallic $T$-dependence;

b) Resistance at peak $\sim h/e^2$

c) Smooth sign-changing $\rho_{xy}$ no plateau;

d) Quasi-plateau in calculated Hall conductivity, double peak in longitudinal conductivity

Novoselov, Geim et al, 2006
Edge transport model

Ideal edge states, contacts with full spin mixing: voltage drop along the edge across each contact universal resistance value

Dissipative edge, unlike conventional QHE!

Backscattering (spin-flips), nonuniversal resistance
Estimate mean free path \( \sim 0.5 \ \mu m \)
Transport coefficients versus filling factor

Broadened, spin-split Landau levels

Bulk conductivity short-circuits edge:

a) peak in \( \rho_{xx} \) at \( \nu = 0 \);
b) smooth \( \rho_{xy} \), sign change, no plateau
c) quasi-plateau in \( G_{xy} = \rho_{xy} / (\rho_{xy}^2 + \rho_{xx}^2) \);
d) double peak in \( G_{xx} = \rho_{xx} / (\rho_{xy}^2 + \rho_{xx}^2) \)

Model explains all general features of the data near \( \nu = 0 \)

The roles of bulk and edge transport interchange (cf. usual QHE):

*longitudinal resistivity due to edge transport, Hall resistivity due to bulk.*
Work in progress: Luttinger liquid on graphene edge?
Quantum Hall Ferromagnetism
Gapless excitations on the edge
The half-integer QHE: Field-Theoretic Parity Anomaly


A novel axial anomaly has been found in gauge theories defined on three-dimensional space-time, which describe dynamics confined to a plane: fermions moving in an external gauge field and governed by the $2 \times 2$ matrix equation (massless Dirac equation)

$$\gamma^\mu (i\partial_\mu - eA_\mu)\Psi = 0$$

(1)

induce a topologically nontrivial vacuum current of abnormal parity,

$$\langle j^\mu \rangle = \pm c \frac{e}{8\pi} \epsilon^{\mu\alpha\beta} F_{\alpha\beta} + \cdots$$

Recall Lorentz-invariant QHE relation

$$j = \sigma_{xy} E, \text{ where } \sigma_{xy} = 1/2$$

(2)

Here $\gamma^\mu$ are three $2 \times 2$ “Dirac” matrices (Pauli matrices) and $A_\mu$ is the external vector potential, leading to the field strength $F_{\alpha\beta}$.

c=1 for Abelian gauge field
Anomaly: relation to fractional quantum numbers

The purpose of this paper is to derive similar results in the three-dimensional case under present discussion. We show that for static background fields in the $A_0=0$ Weyl gauge, the Dirac Hamiltonian corresponding to (1) possesses a conjugation-symmetric spectrum with zero modes, if the background field satisfies certain requirements. Although the topological interest is mainly in the non-Abelian theory, we shall concern ourselves with the Abelian Maxwell theory, which is of greater physical relevance, since it can describe the motion of charged fermions on a plane perpendicular to an external magnetic $B$ field.

The demonstration is very simple. The Hamiltonian corresponding to (1) is

$$H = \bar{\alpha} \cdot (\bar{p} - e \bar{A}),$$

where the “Dirac” $\alpha$ matrices are the two Pauli matrices: $\alpha^1 = -\sigma^2$, $\alpha^2 = \sigma^1$. The $\beta$ matrix, which would be present if there were a mass term, is taken to be $\sigma^3$. Since $\beta = \sigma^3$ anticommutes with $H$, it serves as a conjugation matrix, and the energy eigenmodes are symmetric about $E = 0$.

$$\bar{\alpha} \cdot (\bar{p} - e \bar{A}) \psi_E = E \psi_E,$$

$$\sigma^3 \psi_E = \psi_{-E}.$$  \hspace{1cm} (4)

Of course in the presence of the mass term, the conjugation symmetry is broken.

To find the zero-energy modes we write the wave function as $\psi_0 = (\psi^0)$, and choose the Coulomb gauge for $\bar{A}$, which we assume to be single valued and well behaved at the origin,

$$A^i = \epsilon^{ij} \partial_j a,$$

$$B = -\nabla^2 a.$$  \hspace{1cm} (5)

Then Eq. (4) reduces to the pair

$$(\partial_x + i \partial_y)u - e(\partial_x + i \partial_y)au = 0,$$

$$(\partial_x - i \partial_y)v + e(\partial_x - i \partial_y)av = 0,$$

with the obvious solution

$$u = \exp(ea) f(x + iy),$$

$$v = \exp(-ea) g(x - iy),$$  \hspace{1cm} (8)

where $f$ and $g$ are arbitrary entire functions. Thus we can form self-conjugate solutions $(\psi^0)$ and $(\psi^0)$. Whether these are acceptable wave functions depends on the large-$r$ behavior of $\psi$. If $a$ grows sufficiently rapidly at large distance, then either $u$ or $v$ will be normalizable, and there exist one or more isolated zero-energy bound states, the multiplicity depending on how many different forms for $f$ or $g$ may be taken.

It is useful to classify the various possibilities in terms of the total flux, which is also proportional to the total induced charge:

$$\langle j^0 \rangle = \pm \frac{e}{4\pi} B,$$

$$Q = \int d^2 \tau \langle j^0 \rangle = \pm \frac{e}{4\pi} \int d^2 \tau \Phi = \pm \frac{e}{\Phi 2},$$ \hspace{1cm} (9)

Each zero-energy state filled (unfilled) contributes $+1/2(-1/2)$ of an electron macroscopically: $(1/2)$*LL density